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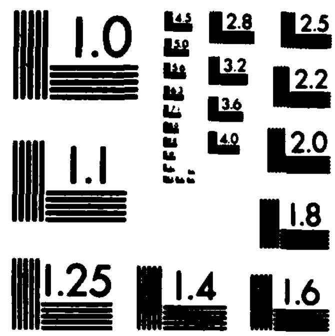
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David W. Taylor Naval Ship Research and Development Center

Bethesda, MD 20084-5000

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Computation, Mathematics and Logistics Department
Research and Development Report

RADSPHERE — A Computer Program for Calculating the Steady-State, Axially Symmetric, Forced Response and Radiation Field of a Submerged Spherical Shell

by

Francis M. Henderson

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A computer program, RADSPHERE, has been developed for calculating the axisymmetric forced vibration response and sound radiation of a thin, elastic, spherical shell, submerged in an infinite fluid. Four cases of excitation are provided: (1) a concentrated radial load acting at a point on the shell surface, (2) two such loads acting simultaneously at antipodal surface points, (3) a uniform load distributed over a sector of the surface, and (4) two sector loads acting simultaneously with respect to an axis of the sphere. The calculated quantities are the shell surface radial velocity and the acoustic pressure at both the shell surface and in the far-field. RADSPHERE results are generated in tabular form and in graphic form, the latter through use of a companion program PLOTTER which provides an interface with the facilities of the software package DISSPLA.

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ABSTRACT

A computer program, RADSPHERE, has been developed for calculating the axisymmetric forced vibration response and sound radiation of a thin, elastic, spherical shell, submerged in an infinite fluid. Four cases of excitation are provided: (1) a concentrated radial load acting at a point on the shell surface, (2) two such loads acting simultaneously at antipodal surface points, (3) a uniform load distributed over a sector of the surface, and (4) two sector loads acting simultaneously with respect to an axis of the sphere. The calculated quantitites are the shell surface radial velocity and the acoustic pressure at both the shell surface and in the far-field. RADSPHERE results are generated in tabular form and in graphic form, the latter through use of a companion program PLOTTER which provides an interface with the facilities of the software package DISSPLA.

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MATHEMATICAL FORMULATION

RADSPHERE was developed in response to the need to calculate analytic solutions for validating a finite element capability called NASHUA (NAStran Helmholtz Underwater Acoustics, Everstine et al.[1]), currently under development.

The formulations which the program evaluates are taken either directly or are derived from the analysis of axisymmetric forced vibration of submerged thin spherical shells by Junger and Feit[2]. To facilitate referencing, exact mathematical expressions taken from this source will be designated by the notations J & F with the original equation numbers, in addition to the numbering required in the present report.

A convenient starting point for the analysis to be presented here is the expression for forced radial velocity response of the shell in vacuo (J&F, Eq. 10.10)

$$\dot{w}(\theta) = \sum_{n=0} \frac{f_n}{Z_n} P_n(\cos\theta) \tag{1}$$

where θ = the colatitude or polar angle with respect to an axis of the sphere

$P_n(\cos\theta)$ = Legendre polynomial of order n and degree 1

Z_n = in-vacuo modal impedance of the shell

f_n = coefficients in the Legendre series expansion for an applied load acting on the shell.

The in-vacuo modal impedance is given by (J&F, Eq. 10.11),

$$Z_n = -i\frac{h}{a}\rho_s c_p \frac{[\Omega^2-(\Omega_n^{(1)})^2]\ [\Omega^2-(\Omega_n^{(2)})^2]}{\Omega^3-\Omega(n^2+n-1+\nu)} \tag{2}$$

where $i = \sqrt{-1}$

h = shell thickness

a = shell radius

ρ_s = mass density of the shell material

ν = Poisson's ratio

$c_p = \left[\frac{E}{\rho_s(1-\nu^2)}\right]^{1/2}$ with E denoting Young's modulus

Ω = dimensionless frequency of vibration

$= \frac{\omega a}{c_p} = \left(\frac{c}{c_p}\right) ka$ with ω = frequency (rad/s)

c = speed of sound in the fluid

k = wave number = ω/c

$\Omega_n^{(1)}$, $\Omega_n^{(2)}$, = dimensionless frequencies of the in-vacuo shell lower and upper branch modes n, respectively.

The total loading on the shell is expressed as a combination of the driving force and radiation load as

$$p(\theta)_{total} = p(\theta)_{applied} + p(\theta)_{rad}. \tag{3}$$

For axisymmetric modes (J&F, Eq. 10.1)

$$p(\theta)_{rad} = -\sum_{n=o} z_n \dot{W}_n P_n\ (\cos\ \theta). \tag{4}$$

Here z_n is the specific acoustic impedance given by

$$z_n = i\rho_{fluid} c \frac{h_n(ka)}{h_n'(ka)}$$

where ρ_{fluid} = mass density of fluid

$h_n(ka)$ = a spherical Hankel function of first kind and order n with argument ka,

$\dot{W}_n$ = the modal velocity amplitude.

The analysis now requires that the applied load $p(\theta)_{applied}$ be expanded in a Legendre series which can then be combined with the series in Eq. 4, to yield the coefficients f_n of the expansion of the total load

$$p(\theta)_{total} = \sum_{n=o} f_n P_n (\cos \theta). \tag{5}$$

POINT LOAD

The first applied loading to be considered here is illustrated in Fig. 1. In this figure F_o denotes the force magnitude and t is the time variable. The direction of the force indicated is positive outward toward the fluid. The force is represented[3] with the Dirac delta function

$$F (\cos \theta) = F_o \frac{\delta(\cos \theta - 1)}{2\pi a^2} \tag{6}$$

Let the Legendre expansion of this force be represented by

$$F (\cos \theta) = \sum_m \mathcal{f}_m P_m (\cos \theta) \tag{7}$$

where $\mathcal{f}_m$ denotes the expansion coefficients.

Then,

$$F (\cos \theta) P_n (\cos \theta) = \sum_m \mathcal{f}_m P_m (\cos \theta) P_n (\cos \theta). \tag{8}$$

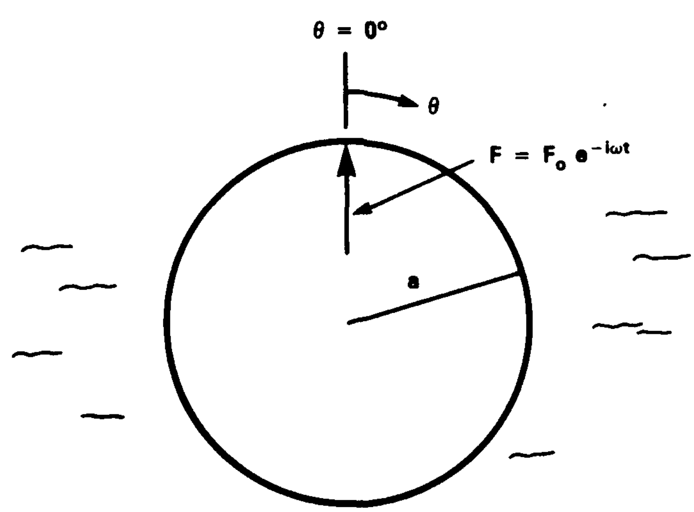

Fig. 1. Concentrated load acting on a submerged spherical shell.

Integrating Eq. 8 yields

$$\int_{-1}^{1} F(\cos\theta)\, P_n(\cos\theta)\, d(\cos\theta) = \int_{-1}^{1} \sum_m f_m P_m(\cos\theta)\, P_n(\cos\theta)\, d(\cos\theta)$$

$$= \sum_m f_m \int_{-1}^{1} P_m(\cos\theta)\, P_n(\cos\theta)\, d(\cos\theta)$$

$$= f_n \frac{2}{2n+1} \qquad (9)$$

utilizing the orthogonality relations given by Wylie[4]

$$\int_{-1}^{1} P_m(\cos\theta)\, P_n(\cos\theta)\, d(\cos\theta) = \begin{cases} 0, & m \neq n \\ \dfrac{2}{2n+1}, & m = n \end{cases} \qquad (10)$$

Returning to the left side of Eq. 9 there results

$$\int_{-1}^{1} F(\cos\theta)\, P_n(\cos\theta)\, d(\cos\theta) = \frac{F_o}{2\pi a^2} \int_{-1}^{1} \delta(\cos\theta - 1)\, P_n(\cos\theta)\, d(\cos\theta)$$

$$= \frac{F_o}{2\pi a^2} P_n(1) = \frac{F_o}{2\pi a^2} \qquad (11)$$

when utilizing the fact that[4],

$$\int_{-\infty}^{\infty} f(x)\, \delta(x-x_o)\, dx = f(x_o) \tag{12}$$

and the identity $P_n(1) = 1$ for all n.

From Eqs. 9 and 11 one obtains

$$\mathcal{f}_n \frac{2}{2n+1} = \frac{F_o}{2\pi a^2} \tag{13}$$

from which

$$\mathcal{f}_n = \frac{(2n+1)F_o}{4\pi a^2} . \tag{14}$$

Combining the coefficients $\mathcal{f}_n$ with those of Eq. 4 yields the coefficients f_n of the total load for this case (J&F, Eq. 10.9)

$$f_n = \frac{(2n+1)\ F_o}{4\pi a^2} - z_n \dot{W}_n . \tag{15}$$

In view of the fact that in Eq. 1 $f_n/Z_n = \dot{W}_n$, one has by substituting for f_n from Eq. 15

$$\dot{W}_n = \frac{\dfrac{(2n+1)F_o}{4\pi a^2} - z_n \dot{W}_n}{Z_n} . \tag{16}$$

This can be solved for $\dot{W}_n$ (J&F, Eq. 10.12), obtaining

$$\dot{W}_n = \frac{(2n+1)\ F_o}{4\pi a^2 (Z_n + z_n)} . \tag{17}$$

Substituting Eq. 17 into Eq. 1 gives the shell radial velocity for the submerged case. (J&F, Eq. 10.13)

$$\dot{w}(\theta) = \frac{F_o}{4\pi a^2} \sum_n \frac{(2n+1)}{Z_n + z_n} P_n\ (\cos\theta). \tag{18}$$

Following Junger and Feit's[2] analysis, Eq. 18 is then multipled by $i\rho_{fluid} c h_n(kR)/h_n'(ka)$ to obtain the resulting sound pressure for field points at $(R,\ \theta)$ where R is the radius from the center of the sphere. If R=a, the multiplying factor becomes z_n, Eq. 4, and one has the expression

$$p(a,\theta) = \frac{F_o}{4\pi a^2} \sum_n \frac{(2n+1)\ z_n}{Z_n + z_n} P_n\ (\cos\theta). \tag{19}$$

for the shell surface pressure.

For the far-field pressure, $h_n(kR)$ in the multiplying factor is approximated (J&F, Eq. 8.11c) by

$$h_n(kR) \approx \frac{1}{kR} e^{ikR} e^{-i\left(\frac{n+1}{2}\pi\right)},\ kR >> n^2+1. \tag{20}$$

It is readily verified that

$$i \frac{e^{ikR}}{kR} e^{-i\left(\frac{n+1}{2}\pi\right)} = (-i)^n. \tag{21}$$

Hence, the multiplying factor is approximated by

$$i\rho_{fluid} c h_n(kR)/h_n'(ka) \approx \left[\left(\frac{e^{ikR}}{kR}\, \rho_{fluid} c\right) \Big/ h_n'(ka)\right] (-i)^n \tag{22}$$

and the far-field formulation corresponding to Eq. 19 is then (J&F Eq. 10.14)

$$p(R,\theta) = \frac{F_o \rho_{fluid} c\ e^{ikR}}{4\pi a^2 kR} \sum_n \frac{(-i)^n (2n+1)}{(Z_n + z_n) h_n'(ka)} P_n\ (\cos\theta),\ kR >> 1. \tag{23}$$

Equations 18, 19, and 23 are the formulations evaluated by RADSPHERE for the case of a concentrated harmonic force acting at the apex of the polar axis.

SECTOR LOAD

The second applied loading of interest is illustrated in Fig. 2. This load is defined by

$$F\ (\cos\theta) = \begin{cases} F_o,\ o \le \theta \le \alpha \le \pi \\ o,\ \text{otherwise.} \end{cases} \tag{24}$$

For this case the left side of Eq. 9 is

$$\int_{-1}^{1} F\ (\cos\theta)\ P_n\ (\cos\theta)\ d\ (\cos\theta) = F_o \int_{\cos\alpha}^{1} P_n\ (\cos\theta)\ d\ (\cos\theta). \tag{25}$$

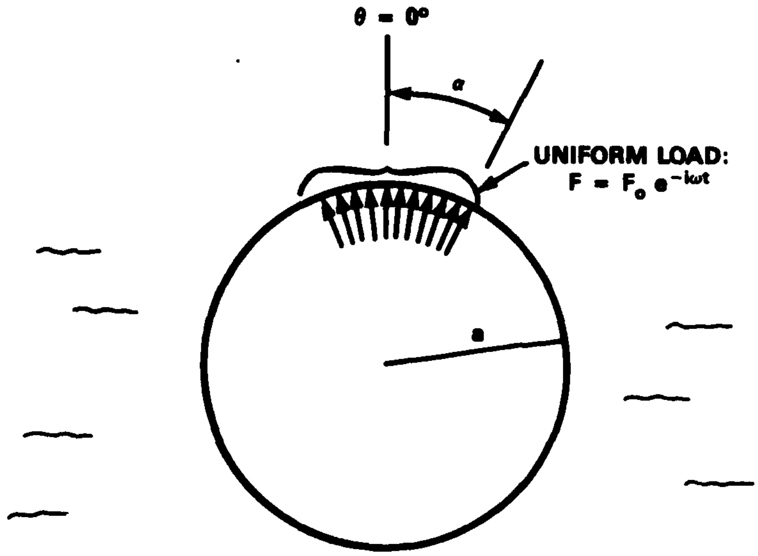

Fig. 2. Uniform load acting over a polar sector of a submerged spherical shell.

Utilizing the identity[4],

$$P'_{n+1}(x) - P'_{n-1}(x) = (2n+1)\, P_n(x), \tag{26}$$

Eq. 25 yields

$$F_o \int_{\cos\alpha}^{1} P_n(\cos\theta)\, d(\cos\theta) = \frac{F_o}{2n+1} \int_{\cos\alpha}^{1} [P'_{n+1}(\cos\theta) - P'_{n-1}(\cos\theta)]\, d(\cos\theta)$$

$$= \frac{F_o}{2n+1} [P_{n+1}(\cos\theta) - P_{n-1}(\cos\theta)]_{\cos\alpha}^{1}$$

$$= \frac{F_o}{2n+1} \{1 - 1 - [P_{n+1}(\cos\alpha) - P_{n-1}(\cos\alpha)]\}$$

$$= \frac{F_o}{2n+1} [P_{n-1}(\cos \alpha)-P_{n+1}(\cos \alpha)] \tag{27}$$

with the identity for $P_n(1)$ given on page 6.

Equating this result to the right side of Eq. 9 and solving for $\mathcal{f}_n$ leads to

$$\mathcal{f}_n = \frac{F_o}{2} [P_{n-1} (\cos \alpha) - P_{n+1} (\cos \alpha)] . \tag{28}$$

Analogous to the steps taken in Eqs. 15-17, respectively, one obtains the total load

$$f_n = \frac{F_o}{2} [P_{n-1} (\cos \alpha)-P_{n+1} (\cos \alpha)] - z_n \dot{W}_n \tag{29}$$

then,

$$\dot{W}_n = \frac{\frac{F_o}{2} [P_{n-1}(\cos \alpha)-P_{n+1}(\cos \alpha)] - z_n \dot{W}_n}{Z_n} \tag{30}$$

and, solving for $\dot{W}_n$,

$$\dot{W}_n = \frac{\frac{F_o}{2} [P_{n-1}(\cos \alpha)-P_{n+1}(\cos \alpha)]}{(Z_n + z_n)} . \tag{31}$$

Substituting $\dot{W}_n$ for (f_n/Z_n) in Eq. 1 gives the shell radial velocity

$$\dot{w}(\theta) = \frac{F_o}{2} \sum_{n=o} \frac{[P_{n-1}(\cos\alpha) - P_{n+1}(\cos\alpha)]}{(Z_n + z_n)} P_n(\cos\theta) \tag{32}$$

from which is obtained, utilizing steps analogous to those used for Eqs. 19 and 23 respectively, the shell surface pressure

$$p(a,\theta) = \frac{F_o}{2} \sum_{n=o} \frac{[P_{n-1}(\cos\alpha) - P_{n+1}(\cos\alpha)]\, z_n}{(Z_n + z_n)} P_n(\cos\theta) \tag{33}$$

and the far-field pressure

$$p(R,\theta) = \frac{F_o \rho_{fluid} c e^{ikR}}{2kR} \sum_{n=o} \frac{(-i)^n [P_{n-1}(\cos\alpha) - P_{n+1}(\cos\alpha)]}{(Z_n + z_n)\, h_n'(ka)} P_n(\cos\theta). \tag{34}$$

Equations 32-34 are the formulations evaluated RADSPHERE for the case of an harmonic driving force distributed uniformly over a sector of the shell surface centered on an apex of the polar axis.

The shell response and/or field pressure for two additional load conditions is readily obtained by superposing solutions for the loading conditions just discussed. This process is pictured in Fig. 3. If a complex-valued solution corresponding to load condition I is designated $Sol_I(\theta)$ and for load condition II by $Sol_{II}(\theta)$, then

$$Sol_{II}(\theta_1) = Sol_I(180-\theta_1) \tag{35}$$

and

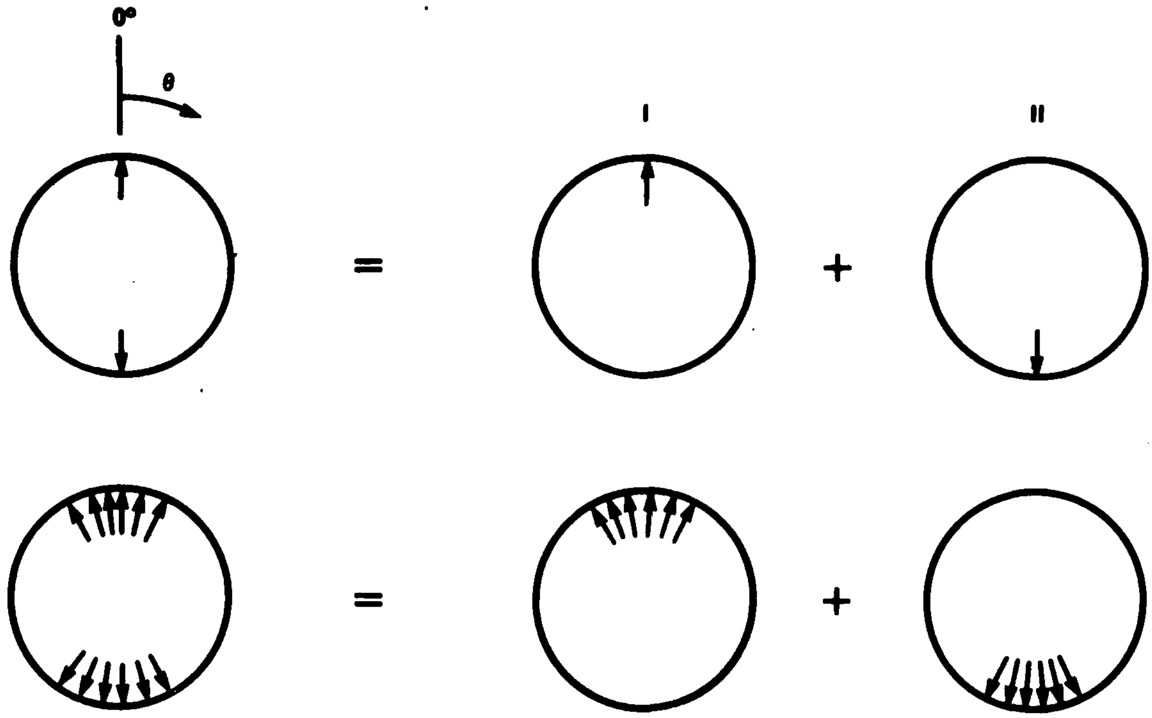

Fig. 3. Symmetric loads synthesized from asymmetric load components (denoted as load conditions I or II).

$$Sol_{sym}\ (\theta_i) = Sol_I\ (\theta_i) + Sol_I\ (180-\theta_i) = Sol_I\ (\theta_i) + Sol_{II}\ (\theta_i). \tag{36}$$

The basic calculations made by RADSPHERE are for load conditions I, which are designated as PROBLEM 1 types in the program data input. The two symmetric conditions are designated as PROBLEM 2 types.

NOTES ON THE COMPUTATION

1. Calculation of the specific acoustic impedance (Eq. 4) requires spherical Hankel functions and their derivatives for particular arguments. In standard notation, the spherical Hankel function h of order n for argument x is represented by

$$h_n(x) = j_n(x) + iy_n(x) \tag{37}$$

where $j_n(x)$ and $y_n(x)$ are the spherical Bessel functions of the first and second kinds, respectively, and of order n. To obtain these latter two functions, IMSL[5] subroutines MMBSJR and MMBSYN are first used to calculate $J_{n+1/2}(x)$ and $Y_{n+1/2}(x)$ which are the Bessel functions of the first and second kinds, respectively, and of fractional order. The spherical Hankel functions are then calculated (Abramowitz and Stegun[6]) by the rule

$$h_n(x) = \sqrt{\pi/2x}\,[J_{n+1/2}(x) + iY_{n+1/2}(x)],\ n=0,1,2,\ \ldots \tag{38}$$

Derivatives of these functions are obtained[6] by the rule

$$h_n'(x) = \frac{1}{2n+1}\,[nh_{n-1}(x) - (n+1)h_{n+1}(x)],\ n=0,1,2,\ \ldots \tag{39}$$

2. Evaluation of the Legendre polynomials of order n and degree $\nu=1$, for argument x, are obtained by the recursion[7]

$$P_o(x) = 1,$$

$$P_{n+1}(x) = \frac{2n+1}{n+1}\,x\,P_n(x) - \frac{n}{n+1}\,P_{n-1}(x). \tag{40}$$

3. The RADSPHERE program requires specification of the maximum number of terms that will be included in any series being summed (surface pressure, surface velocity, field pressure) for a particular application. The purpose of this is to bound the computing effort that will be expended trying to obtain convergence in the event of

very small magnitudes (i.e., nodes in θ profiles for velocity or pressure). Below this maximum number, the series summation is terminated when the condition

$$\frac{\left| Sum_{(n+1)} - Sum_n \right|}{\left| Sum_n \right|} \leq \varepsilon \qquad (41)$$

is met, with ε being a specified error bound. The index n references only nonzero terms in the series being summed.

4. The RADSPHERE program treats the frequency range of interest (in terms of ka) as divided into subintervals i, not necessarily adjacent, each having uniform spacing (Δka) within. This treatment facilitates computing for a set of nonuniformly spaced ka's (since an "interval" can consist of a single ka) as well as computing at varying densities of ka over the range of interest. The latter capability is generally useful in the case of searching for, or refining resonant frequencies of the submerged shell.

DATA INPUT TO RADSPHERE

Data for RADSPHERE is input with list-directed (unformatted) records.[8] Some of the data control option selections and/or branching within the program. These parameters are designated by their FORTRAN variable names. The data are presented in Table 1 in the order required by the program. In this table and all subsequent tables throughout the report, each line in the column headed "Data" represents one card record of data.

Table 1. RADSPHERE data input.

Data	Description	Application	Value
Title	$\leq$ 50 alphanumeric characters in card columns 1-50	All	
NFLAGF	Load selection	All	= 1 (point drive at $\theta=0°$ (Fig. 1)) = 2 (sector drive (Fig. 2))
F_o	Magnitude of load	All	
α	Polar angle (deg) of sector load (Fig. 2)	When NFLAGF=2	
a	Shell radius	All	
h	Shell thickness	All	
ρ_s	Shell mass density	All	
ν	Poisson's ratio	All	
E	Young's modulus	All	
η	Structural loss factor	All	= 0, or a specified value
ρ_{fluid}	Fluid mass density	All	
c	Speed of sound in the fluid	All	
NDIAGN	Controls printing of intermediate quantities for check purposes	All	= 1 (Do not print) = 2 (Print)
NPROB	Selector for printing calculated results	All	= 1 (Problem type 1 (Fig. 3) results only) = 2 (Problem type 2 (Fig. 3) results only) = 3 (Both problem types 1 and 2, results)

Table 1. (Continued)

Data	Description	Application	Value
NFLAG	Selector for quantities to be calculated		= 1 (shell surface pressure) = 2 (shell surface and field pressure) = 3 (field pressure) = 4 (shell surface pressure and radial velocity) = 5 (shell surface radial velocity)
NRADII	Number of radii at which field calculations are to be made, maximum = 10	When NFLAG = 2 or 3	
R_1 R_2 . . . R_{NRADII}	Distinct radii from center of shell to field points.	When NFLAG = 2 or 3	
NTERMS	Bound on no. of terms in any series to be computed (see pp. 13-14), maximum = 200	All	
ε	Error criterion for convergence of series summations.	All	
θ_1	Initial colatitude (deg) for shell surface and/or field points at which calculations are to be made (see Fig. 3).	All	
θ_2	Final colatitude (deg)	All	
$\Delta\theta$	Increment in colatitude (deg) Max $\left[\frac{\theta_2-\theta_1}{\Delta\theta} + 1\right] = 91$	All	
NINT	Number of intervals in ka (see page 14)	All	

Table 1. (Continued)

Data	Description	Application	Value
$(ka_1)_1$, $(ka_2)_1$, $(\Delta ka)_1$ $(ka_1)_2$, $(ka_2)_2$, $(\Delta ka)_2$ . . . . $(ka_1)_{NINT}$, $(ka_2)_{NINT}$, $(\Delta ka)_{NINT}$	Initial ka, final ka, Δka for the i^{th} interval in ka.	All	

With regard to the data pertaining to the shell geometry and the shell material and fluid properties, it is noted that any system of units may be used so long as consistency is maintained.

The data flag NDIAGN is used to obtain printout of the following quantities for diagnostic purposes:

a. $\left[\Omega_n^{(1)}\right]^2$, $\left[\Omega_n^{(2)}\right]^2$ (see p. 3)

b. P_n (cos α) (see Eq. 28)

c. $J_{n+1/2}$ (ka), $Y_{n+1/2}$ (ka) (see p. 13)

d. h_n (ka) (see Eq. 38)

e. h_n' (ka) (see Eq. 39)

f. z_n (see p. 3)

g. Z_n (see p. 2)

h. $\dfrac{z_n}{(Z_n+z_n)}$ (see Eq. 19)

i. $\dfrac{1}{(Z_n+z_n)\ h_n'\ (ka)}$ (see Eq. 23)

j. $\dfrac{1}{(Z_n+z_n)}$ (see Eq. 18)

k. P_n (cos θ) (see Eq. 1)

The NDIAGN is also used to echo the data written to plot files.

To prevent the "wallpaper" effect when using NDIAGN, it is advisable to trim diagnostic runs to the minimum number of ka's (ideally, = 1) and as low a value of "NTERMS" as possible.

The loss factor η is introduced into the calculations via the familiar mechanism[9] of scaling E by $(1-i\eta)$. The sign of $i\eta$ is consistent with the exponential form for the time function $e^{-i\omega t}$ assumed in the analysis by Junger and Feit.[2]

The RADSPHERE program is dimensioned for a maximum of 91 values of colatitude for computations of response profiles, and a maximum of 2001 values of ka per interval of ka. The number of ka intervals is arbitrary.

For calculations involving only a single colatitude, NPROB must be assigned the value of one. This restriction results from the fact that the single colatitude case was designed exclusively to facilitate searching for resonant ka. While either single or double pole excitation could be used for this case (see Fig. 3), only the single excitation response is efficient to calculate because only a single solution is required (see Eq. 36).

DATA INPUT TO PLOTTER

Certain subsets of the results computed by RADSPHERE can be obtained in graph form via the companion program, PLOTTER, which utilizes the software package DISSPLA[10]. In particular, one can obtain polar plots of shell surface pressure and radial velocity, as well as far-field pressure, for each ka, or log-linear plots of the same quantities at a single colatitude over a range of ka values.

Data to be plotted are written by RADSPHERE on a permanent file device TAPE12. Following a RADSPHERE run this file must be cataloged for subsequent use by PLOTTER when plots are desired. Before data intended for polar plots are written on

TAPE12, a subroutine, SORT, recasts the data from the system used in computing (see Fig. 3) to the coordinate system used for plotting, Fig. 4. In the plotting coordinate system, a result computed at colatitude $\theta = 0°$ appears at $\theta = 90°$; a result at colatitude $\theta = 90°$ appears at $\theta = 0°$; a result at colatitude $\theta = 180°$ appears at $\theta = 270°$; etc. Subroutine SORT also augments (via axial symmetry) the original data computed in the colatitude range $0° \leq \theta \leq 180°$ to the full range in plotting coordinates, $0° \leq \theta \leq 360°$.

For the variables shell surface pressure and field pressure, the quantity plotted is always the absolute value. For log-linear plots of shell surface velocity versus ka, the plotted quantity is also the absolute value. However, for polar plots of velocity profile, it seemed advisable to attempt to incorporate the effect of phase. An approach is to plot

$$\dot{w}(\theta) = \sin\gamma \cdot \left|\dot{w}(\theta)\right| = \mathcal{I}\,\dot{w}(\theta), \tag{42}$$

where $\gamma = \tan^{-1}\left[\dfrac{\mathcal{I}\dot{w}(\theta)}{\mathcal{R}\dot{w}(\theta)}\right]$

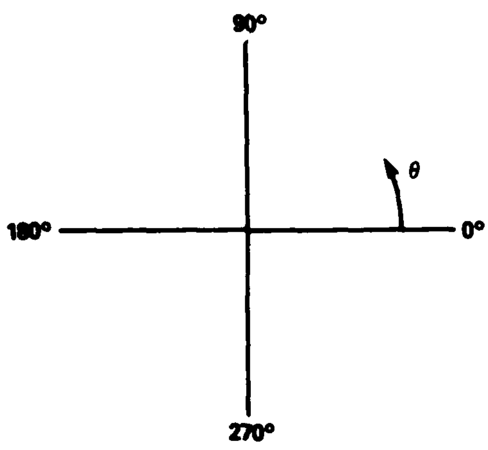

Fig. 4. Coordinate system for plotting.

with $\mathcal{J}$, $\mathcal{R}$ designating, respectively, imaginary and real parts of $\dot{w}(\theta)$. The choice of sin γ was made to obtain relative maxima of response at $\theta = 90°$ and $\theta = 270°$, as induced by the excitation force(s).

Much of the input data required by PLOTTER (aside from the plot data) is supplied on the plot file from RADSPHERE. The remaining input data required is largely that necessary to provide the user considerable flexibility in shaping the plots.

The following set of data (Table 2) is provided for each interval of ka computed.

Table 2. Plotter data input.

Data	Description	Application
NKA_i	Total ka computed for the i^{th} interval. (Available from RADSPHERE printout.) Maximum = 2001.	Polar and log-linear plots.

ORIG,RMAX,RSTEP	Minimum value, maximum value, and step size for the radial direction. Step size is determined by the fraction, RMAX/5.
	
	Note: This triad of data is provided for each ka in the i^{th} interval for each variable (computed) according to the following order:
	
	Shell Surface Pressure
	  Problem type 1
	  Problem type 2
	
	Field Pressure at
	  First Radius
	    Problem type 1
	    Problem type 2
	
	  Second Radius
	    Problem type 1
	    Problem type 2
	  .
	  .
	  .
		Polar plots.

Table 2. (Continued)

Data	Description	Application
	Shell Surface Velocity Problem type 1 Problem type 2	
NCYCLE, YMIN	Number of cycles and minimum value for the y axis.	
XSTEP	Step size in the x-direction. Note: x_{min} + 10*XSTEP = x_{max}.	
NXTICK,NYTICK	Parameters controlling the placement of grid lines in the x- and y-directions, respectively. For the purpose here, the particular form of these parameters is NXTICK = -m, NYTICK = -n, which places grid lines at every m^{th} and n^{th} subdivision on the respective axes.	Log-linear plots.
	Note: This group of three cards is provided in each ka interval for each computed variable in the same order as listed above for polar plots. Only type 1 problem data will be present as previously stated.	

All data for PLOTTER is specified in list-directed form[8].

The output from PLOTTER consists of a single page of printed output which echos identifying data from the RADSPHERE data file, and PLFILE, written by DISSPLA. The latter file is cataloged for postprocessing either to a Tektronix screen, interactively, or to a tape for off-line Calcomp plots.

CALCULATIONS

The final section of this report presents several calculations to illustrate some of the features of RADSPHERE that have been discussed and to provide examples of the various forms of edited output.

UNIFORMLY PULSATING SPHERICAL SHELL

For this problem (see Fig. 5), the submerged shell is driven by a uniformly distributed pressure of magnitude F_o acting with harmonic time variance over the interior shell surface. This is an ideal problem from the standpoint of validating RADSPHERE, because closed form solutions can be derived[1] for the shell surface pressure and radial velocity and the field pressure. Figures 2 and 3 show that this problem can be cast in the form of superposed loadings in which I and II are sector loads with polar angle, $\alpha = 90°$. In RADSPHERE terminology, this is a Problem 2 type example. The calculation is performed in two runs: a. to obtain the surface pressure and radial velocity, and b. to obtain field pressure. The data input is shown in Table 3. Data for the shell are in MKS units for steel.

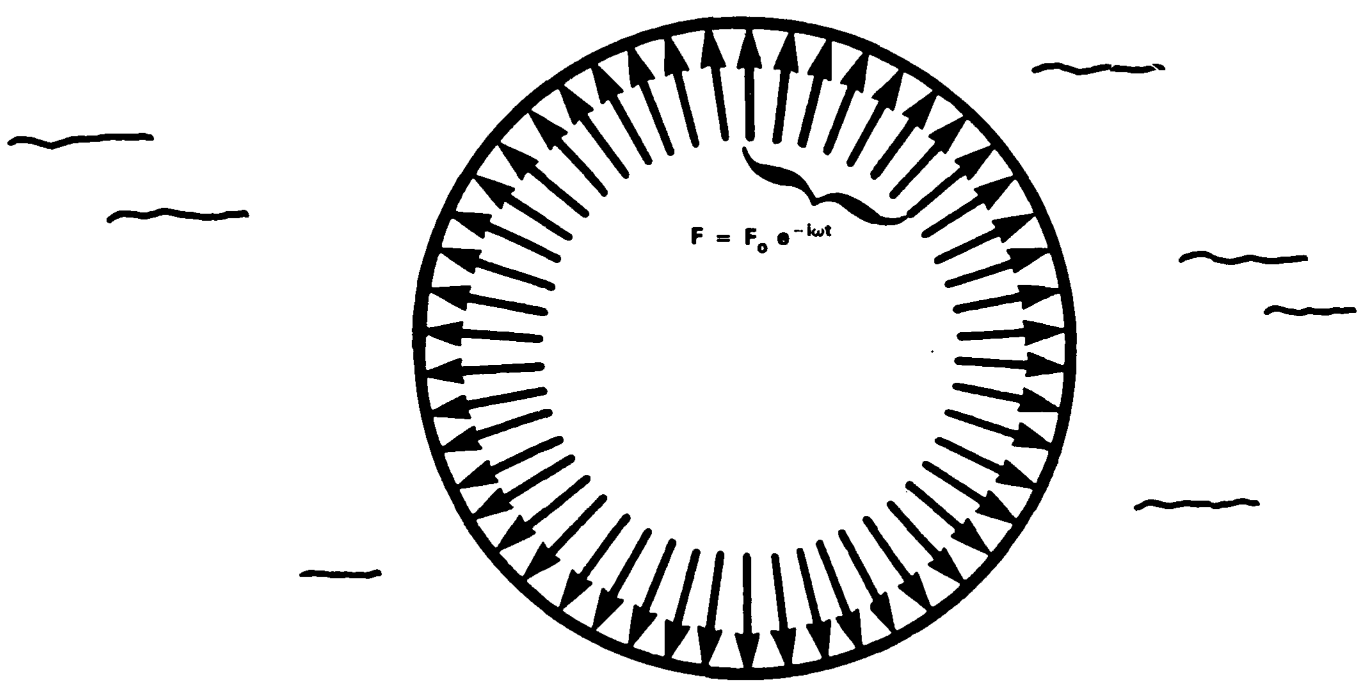

Fig. 5. Submerged uniformly pulsating spherical shell.

Table 3. RADSPHERE input data for problem of uniformly pulsating spherical shell.

Problem Part	Data Item	Value
Part a, to calculate surface pressure and radial velocity.	Title	Uniformly Pulsating Sphere
	NFLAGF	2
	F_o	1.
	α	90.
	a	5.
	h	0.15
	ρ_s	7669.
	ν	0.3
	E	2.07E11
	η	0.
	ρ_{fluid}	1000.
	c	1524.
	NDIAGN	1
	NPROB	2
	NFLAG	4
	NTERMS	30
	ε	0.0001
	θ_1	0.
	θ_2	180.
	$\Delta\theta$	10.
	NINT	1
	$(ka_1)_1$, $(ka_2)_1$, $(\Delta ka)_1$	5., 5., 1.
Part b, to calculate field pressure	Title . . . NPROB	same as for a.
	NFLAG	3
	NRADII	1
	R_1	100.

Table 3. (Continued)

Problem Part	Data Item	Value
	NTERMS . . . $(ka_1)_1$, $(ka_2)_1$, $(\Delta ka)_1$	same as for a.

The respective printouts from RADSPHERE for these sets of data are shown in Fig. 6 (Part a) and Fig. 7 (Part b). Referring to the tabular output in these figures, the columns headed ERROR 1 and ERROR 2 give the error achieved at convergence for components I and II, respectively (see Fig. 3) of the solution. The columns headed FLAG 1 and FLAG 2 give the number of series terms required to achieve the respective levels of convergence and thus are indicators of the degree of difficulty in obtaining the solution at each colatitude.

```
                    D    A    T    A         E    C    H    O

PROBLEM TITLE - UNIFORMLY PULSATING SPHERE

DATA FOR SHELL

     RADIUS          =5.00
     THICKNESS       = .15
     MASS DENSITY    =7.6690E+03
     POISSON RATIO   = .30
     YOUNG S  MODULUS=2.0700E+11
     LOSS FACTOR     =0.00

DATA FOR EXCITATION

     SECTOR LOAD
          POLAR ANGLE = 90.00  DEGREES
          MAGNITUDE   =  1.00

DATA FOR FLUID

     MASS DENSITY   =1000.00
     SPEED OF SOUND =1524.00

PROBLEM TYPES TO BE CALCULATED

               2

QUANTITIES TO BE COMPUTED

     SHELL SURFACE            SHELL SURFACE           FAR - FIELD
        VELOCITY                 PRESSURE               PRESSURE

           X                        X

DIAGNOSTICS

     DO NOT PRINT

CALCULATIONS TO BE MADE FOR -

     INITIAL COLATITUDE =  0.00
     FINAL COLATITUDE   =180.00
     DELTA COLATITUDE   = 10.00

MAX. NO. OF TERMS FOR A SERIES = 30

CONVERGENCE CRITERION = 1.0000E-04

KA INTERVAL NUMBER IS    1

   INITIAL KA = 5.0000E+00
   FINAL KA   = 5.0000E+00
   DELTA KA   = 1.0000E+00
```

Fig. 6. RADSPHERE output for uniformly pulsating spherical shell (Part a).

PRESSURE ON SURFACE OF SPHERE

PROBLEM TITLE - UNIFORMLY PULSATING SPHERE
PROBLEM 2. - OUTWARD DISTRIBUTED FORCE AT BOTH NORTH AND SOUTH POLES.

KA= 5.00

FREQUENCY = 242.55 HZ

COLATITUDE(DEGREES)	PRESSURE		ABSOLUTE VALUE	PHASE(DEG)	ERROR 1	ERROR 2	FLAG 1	FLAG 2
0.00	9.2708E-01	-3.7857E-01	1.0014E+00	337.79	1.94E-09	2.79E-09	3	3
10.00	9.2708E-01	-3.7857E-01	1.0014E+00	337.79	1.86E-09	2.66E-09	3	3
20.00	9.2708E-01	-3.7857E-01	1.0014E+00	337.79	1.63E-09	2.30E-09	3	3
30.00	9.2708E-01	-3.7857E-01	1.0014E+00	337.79	1.26E-09	1.74E-09	3	3
40.00	9.2708E-01	-3.7857E-01	1.0014E+00	337.79	7.84E-10	1.05E-09	3	3
50.00	9.2708E-01	-3.7857E-01	1.0014E+00	337.79	2.55E-10	3.28E-10	3	3
60.00	9.2708E-01	-3.7857E-01	1.0014E+00	337.79	2.76E-10	3.37E-10	3	3
70.00	9.2708E-01	-3.7857E-01	1.0014E+00	337.79	7.44E-10	8.57E-10	3	3
80.00	9.2708E-01	-3.7857E-01	1.0014E+00	337.79	1.08E-09	1.17E-09	3	3
90.00	9.2708E-01	-3.7857E-01	1.0014E+00	337.79	1.24E-09	1.24E-09	3	3
100.00	9.2708E-01	-3.7857E-01	1.0014E+00	337.79	1.17E-09	1.08E-09	3	3
110.00	9.2708E-01	-3.7857E-01	1.0014E+00	337.79	8.57E-10	7.44E-10	3	3
120.00	9.2708E-01	-3.7857E-01	1.0014E+00	337.79	3.37E-10	2.76E-10	3	3
130.00	9.2708E-01	-3.7857E-01	1.0014E+00	337.79	3.28E-10	2.55E-10	3	3
140.00	9.2708E-01	-3.7857E-01	1.0014E+00	337.79	1.05E-09	7.84E-10	3	3
150.00	9.2708E-01	-3.7857E-01	1.0014E+00	337.79	1.74E-09	1.26E-09	3	3
160.00	9.2708E-01	-3.7857E-01	1.0014E+00	337.79	2.30E-09	1.63E-09	3	3
170.00	9.2708E-01	-3.7857E-01	1.0014E+00	337.79	2.66E-09	1.86E-09	3	3
180.00	9.2708E-01	-3.7857E-01	1.0014E+00	337.79	2.79E-09	1.94E-09	3	3

Fig. 6. (Continued)

S H E L L   S U R F A C E   R A D I A L   V E L O C I T Y

PROBLEM TITLE - UNIFORMLY PULSATING SPHERE
PROBLEM 2. - OUTWARD DISTRIBUTED FORCE AT BOTH NORTH AND SOUTH POLES.

KA= 5.00

FREQUENCY = 242.55 HZ

COLATITUDE(DEGREES)	VELOCITY		ABSOLUTE VALUE	PHASE(DEG)	ERROR 1	ERROR 2	FLAG 1	FLAG 2
0.00	6.5800E-07	-1.2674E-07	6.7010E-07	349.10	1.74E-09	2.51E-09	3	3
10.00	6.5800E-07	-1.2674E-07	6.7010E-07	349.10	1.67E-09	2.39E-09	3	3
20.00	6.5800E-07	-1.2674E-07	6.7010E-07	349.10	1.46E-09	2.06E-09	3	3
30.00	6.5800E-07	-1.2674E-07	6.7010E-07	349.10	1.13E-09	1.56E-09	3	3
40.00	6.5800E-07	-1.2674E-07	6.7010E-07	349.10	7.02E-10	9.43E-10	3	3
50.00	6.5800E-07	-1.2674E-07	6.7010E-07	349.10	2.28E-10	2.94E-10	3	3
60.00	6.5800E-07	-1.2674E-07	6.7010E-07	349.10	2.47E-10	3.02E-10	3	3
70.00	6.5800E-07	-1.2674E-07	6.7010E-07	349.10	6.65E-10	7.65E-10	3	3
80.00	6.5800E-07	-1.2674E-07	6.7010E-07	349.10	9.69E-10	1.04E-09	3	3
90.00	6.5800E-07	-1.2674E-07	6.7010E-07	349.10	1.11E-09	1.11E-09	3	3
100.00	6.5800E-07	-1.2674E-07	6.7010E-07	349.10	1.04E-09	9.69E-10	3	3
110.00	6.5800E-07	-1.2674E-07	6.7010E-07	349.10	7.65E-10	6.65E-10	3	3
120.00	6.5800E-07	-1.2674E-07	6.7010E-07	349.10	3.02E-10	2.47E-10	3	3
130.00	6.5800E-07	-1.2674E-07	6.7010E-07	349.10	2.94E-10	2.28E-10	3	3
140.00	6.5800E-07	-1.2674E-07	6.7010E-07	349.10	9.43E-10	7.02E-10	3	3
150.00	6.5800E-07	-1.2674E-07	6.7010E-07	349.10	1.56E-09	1.13E-09	3	3
160.00	6.5800E-07	-1.2674E-07	6.7010E-07	349.10	2.06E-09	1.46E-09	3	3
170.00	6.5800E-07	-1.2674E-07	6.7010E-07	349.10	2.39E-09	1.67E-09	3	3
180.00	6.5800E-07	-1.2674E-07	6.7010E-07	349.10	2.51E-09	1.74E-09	3	3

Fig. 6. (Continued)

```
KA VS. SHELL RADIAL VELOCITY AT NORTH POLE.
 PROBLEM 2. - OUTWARD DISTRIBUTED FORCE AT BOTH NORTH AND SOUTH POLES.


    KA          POLAR VELOCITY
 5.0000E+00      6.7010E-07

INTERVAL = 1   TOTAL KA COMPUTED IS    1
```

Fig. 6. (Continued)

D A T A E C H O

PROBLEM TITLE - UNIFORMLY PULSATING SPHERE

DATA FOR SHELL

```
RADIUS          =5.00
THICKNESS       = .15
MASS DENSITY    =7.6690E+03
POISSON RATIO   = .30
YOUNG S  MODULUS=2.0700E+11
LOSS FACTOR     =0.00
```

DATA FOR EXCITATION

```
SECTOR LOAD
     POLAR ANGLE = 90.00  DEGREES
     MAGNITUDE   =  1.00
```

DATA FOR FLUID

```
MASS DENSITY   =1000.00
SPEED OF SOUND =1524.00
```

DATA FOR FIELD

```
NO. OF RADII = 1

     RADIUS( 1) =100.00
```

PROBLEM TYPES TO BE CALCULATED

```
          2
```

QUANTITIES TO BE COMPUTED

SHELL SURFACE VELOCITY	SHELL SURFACE PRESSURE	FAR - FIELD PRESSURE
		X

DIAGNOSTICS

```
DO NOT PRINT
```

CALCULATIONS TO BE MADE FOR -

```
INITIAL COLATITUDE =  0.00
FINAL COLATITUDE   =180.00
DELTA COLATITUDE   = 10.00
```

Fig. 7. RADSPHERE output for uniformly pulsating spherical shell (Part b).

```
MAX. NO. OF TERMS FOR A SERIES = 30

CONVERGENCE CRITERION = 1.0000E-04



KA INTERVAL NUMBER IS    1

   INITIAL KA = 5.0000E+00
   FINAL KA   = 5.0000E+00
   DELTA KA   = 1.0000E+00
```

Fig. 7. (Continued)

FIELD PRESSURE AT RADIUS = 100.00

PROBLEM TITLE - UNIFORMLY PULSATING SPHERE
PROBLEM 2. - OUTWARD DISTRIBUTED FORCE AT BOTH NORTH AND SOUTH POLES.

KA= 5.00

FREQUENCY = 242.55 HZ

COLATITUDE(DEGREES)	PRESSURE		ABSOLUTE VALUE	PHASE(DEG)	ERROR 1	ERROR 2	FLAG 1	FLAG 2
0.00	4.6780E-02	1.7851E-02	5.0070E-02	20.89	1.93E-09	2.38E-09	3	3
10.00	4.6780E-02	1.7851E-02	5.0070E-02	20.89	1.85E-09	2.28E-09	3	3
20.00	4.6780E-02	1.7851E-02	5.0070E-02	20.89	1.62E-09	1.97E-09	3	3
30.00	4.6780E-02	1.7851E-02	5.0070E-02	20.89	1.24E-09	1.50E-09	3	3
40.00	4.6780E-02	1.7851E-02	5.0070E-02	20.89	7.74E-10	9.18E-10	3	3
50.00	4.6780E-02	1.7851E-02	5.0070E-02	20.89	2.50E-10	2.90E-10	3	3
60.00	4.6780E-02	1.7851E-02	5.0070E-02	20.89	2.68E-10	3.02E-10	3	3
70.00	4.6780E-02	1.7851E-02	5.0070E-02	20.89	7.17E-10	7.79E-10	3	3
80.00	4.6780E-02	1.7851E-02	5.0070E-02	20.89	1.03E-09	1.08E-09	3	3
90.00	4.6780E-02	1.7851E-02	5.0070E-02	20.89	1.16E-09	1.16E-09	3	3
100.00	4.6780E-02	1.7851E-02	5.0070E-02	20.89	1.08E-09	1.03E-09	3	3
110.00	4.6780E-02	1.7851E-02	5.0070E-02	20.89	7.79E-10	7.17E-10	3	3
120.00	4.6780E-02	1.7851E-02	5.0070E-02	20.89	3.02E-10	2.68E-10	3	3
130.00	4.6780E-02	1.7851E-02	5.0070E-02	20.89	2.90E-10	2.50E-10	3	3
140.00	4.6780E-02	1.7851E-02	5.0070E-02	20.89	9.18E-10	7.74E-10	3	3
150.00	4.6780E-02	1.7851E-02	5.0070E-02	20.89	1.50E-09	1.24E-09	3	3
160.00	4.6780E-02	1.7851E-02	5.0070E-02	20.89	1.97E-09	1.62E-09	3	3
170.00	4.6780E-02	1.7851E-02	5.0070E-02	20.89	2.28E-09	1.85E-09	3	3
180.00	4.6780E-02	1.7851E-02	5.0070E-02	20.89	2.38E-09	1.93E-09	3	3

INTERVAL = 1   TOTAL KA COMPUTED IS   1

Fig. 7. (Continued)

Table 4 shows the numerical solution to be in excellent agreement with analytic results[1].

Table 4. Comparison of RADSPHERE and analytic results for the uniformly pulsating spherical shell.

	RADSPHERE	Analytic
Shell Surface Radial Velocity	6.5800E-7 - i1.2674E-7	6.58E-7 + i1.27E-7
Shell Surface Pressure	9.2708E-1 - i3.7857E-1	9.27E-1 + i3.79E-1
Field Pressure	4.6780E-2 + i1.7851E-2	4.68E-2 - i1.79E-2

The difference in sign of the imaginary parts reflects the fact that the time dependence for RADSPHERE's analysis is $e^{-i\omega t}$, whereas that for the analytic solution is $e^{i\omega t}$.

POINT-DRIVEN SPHERICAL SHELL

The second calculation is for a spherical shell whose material and geometric properties and excitation force are taken from Hayek's[3] investigation. The physical problem represented here is that shown previously in Fig. 1. The shell surface pressure and radial velocity are computed, and the results graphed in polar form with PLOTTER. The input data for RADSPHERE is given in Table 5.

The value of ka is derived from an expression for the frequency parameter[3], Ω,

$$\Omega^2 = (1/E)\ \rho_s \omega^2\ a^2 \tag{43}$$

with Ω given the value of 0.85. Data for the shell are in in-lb-s units for steel.

Table 5. RADSPHERE input data for problem of point-driven spherical shell.

Data Item	Value
Title	Point-Driven Sphere
NFLAGF	1
F_o	1.
a	5.
h	0.15
ρ_s	0.7347E-3
ν	0.3
E	30.E6
η	0.
ρ_{fluid}	9.580488E-5
c	60000.
NDIAGN	1
NPROB	1
NFLAG	4
NTERMS	90
ϵ	0.0001
θ_1	0.
θ_2	180.
$\Delta\theta$	2.
NINT	1
$(ka_1)_1$, $(ka_2)_1$, $(\Delta ka)_1$	2.86268316, 2.86268316, 1.

RADSPHERE output is shown in Fig. 8. Obviously, from the number of terms required (column "FLAG"), obtaining convergence of velocity at the poles is considerably more difficult than for the corresponsing pressures.

```
D A T A   E C H O

PROBLEM TITLE - POINT-DRIVEN SPHERE

DATA FOR SHELL

     RADIUS            =5.00
     THICKNESS         = .15
     MASS DENSITY      =7.3470E-04
     POISSON RATIO     = .30
     YOUNG S  MODULUS=3.0000E+07
     LOSS FACTOR       =0.00

DATA FOR EXCITATION

     POINT LOAD
          MAGNITUDE   =  1.00

DATA FOR FLUID

     MASS DENSITY   =9.580488E-05
     SPEED OF SOUND =6.000000E+04

PROBLEM TYPES TO BE CALCULATED

               1

QUANTITIES TO BE COMPUTED

     SHELL SURFACE          SHELL SURFACE          FAR - FIELD
        VELOCITY               PRESSURE              PRESSURE

           X                      X

DIAGNOSTICS

     DO NOT PRINT

CALCULATIONS TO BE MADE FOR -

     INITIAL COLATITUDE =  0.00
     FINAL COLATITUDE   =180.00
     DELTA COLATITUDE   =  2.00

MAX. NO. OF TERMS FOR A SERIES = 90

CONVERGENCE CRITERION = 1.0000E-04

KA INTERVAL NUMBER IS   1

   INITIAL KA = 2.8627E+00
   FINAL KA   = 2.8627E+00
   DELTA KA   = 1.0000E+00
```

Fig. 8. RADSPHERE output for point-driven spherical shell.

P R E S S U R E   O N   S U R F A C E   O F   S P H E R E

PROBLEM TITLE - POINT-DRIVEN SPHERE

PROBLEM 1. - OUTWARD CONCENTRATED FORCE AT NORTH POLE.

KA= 2.86

FREQUENCY =5467.32 HZ

COLATITUDE(DEGREES)	PRESSURE		ABSOLUTE VALUE	PHASE(DEG)	ERROR	FLAG
0.00	-3.7092E-01	6.9132E-04	3.7092E-01	179.89	9.99E-05	51
2.00	-3.5746E-01	6.9964E-04	3.5746E-01	179.89	9.43E-05	44
4.00	-3.2066E-01	7.2428E-04	3.2066E-01	179.87	8.43E-05	32
6.00	-2.6707E-01	7.6420E-04	2.6707E-01	179.84	8.90E-05	23
8.00	-1.9988E-01	8.1774E-04	1.9988E-01	179.77	9.88E-05	37
10.00	-1.2804E-01	8.8264E-04	1.2804E-01	179.61	1.44E-05	32
12.00	-5.5180E-02	9.5616E-04	5.5188E-02	179.01	9.13E-05	27
14.00	1.3449E-02	1.0352E-03	1.3488E-02	4.40	6.37E-05	36
16.00	7.5812E-02	1.1163E-03	7.5820E-02	.84	5.07E-05	42
18.00	1.2759E-01	1.1960E-03	1.2760E-01	.54	1.29E-05	28
20.00	1.6783E-01	1.2707E-03	1.6784E-01	.43	2.61E-05	34
22.00	1.9342E-01	1.3369E-03	1.9342E-01	.40	1.81E-05	23
24.00	2.0610E-01	1.3915E-03	2.0610E-01	.39	6.07E-05	29
26.00	2.0384E-01	1.4315E-03	2.0384E-01	.40	4.05E-05	33
28.00	1.8880E-01	1.4544E-03	1.8881E-01	.44	7.79E-06	31
30.00	1.6120E-01	1.4583E-03	1.6120E-01	.52	7.24E-05	17
32.00	1.2529E-01	1.4418E-03	1.2530E-01	.66	4.28E-05	16
34.00	8.4649E-02	1.4039E-03	8.4661E-02	.95	1.83E-05	31
36.00	3.8491E-02	1.3444E-03	3.8514E-02	2.00	7.67E-05	39
38.00	-8.5483E-03	1.2636E-03	8.6412E-03	171.59	4.60E-05	42
40.00	-5.3619E-02	1.1622E-03	5.3631E-02	178.76	7.67E-05	31
42.00	-9.3933E-02	1.0415E-03	9.3938E-02	179.36	6.98E-05	34
44.00	-1.2732E-01	9.0310E-04	1.2732E-01	179.59	2.80E-05	24
46.00	-1.5214E-01	7.4897E-04	1.5214E-01	179.72	6.52E-06	23

Fig. 8. (Continued)

48.00	-1.6703E-01	5.8131E-04	1.6703E-01	179.80	7.71E-05	22
50.00	-1.7000E-01	4.0246E-04	1.7000E-01	179.86	8.20E-05	14
52.00	-1.6556E-01	2.1482E-04	1.6556E-01	179.93	5.12E-05	31
54.00	-1.4967E-01	2.0775E-05	1.4967E-01	179.99	4.71E-06	23
56.00	-1.2449E-01	-1.7795E-04	1.2449E-01	180.08	4.31E-05	19
58.00	-9.3270E-02	-3.7780E-04	9.3271E-02	180.23	4.19E-05	34
60.00	-5.6761E-02	-5.7731E-04	5.6764E-02	180.58	9.86E-05	36
62.00	-1.7643E-02	-7.7535E-04	1.7660E-02	182.52	2.79E-05	23
64.00	2.1300E-02	-9.6994E-04	2.1322E-02	357.39	7.94E-06	42
66.00	5.8085E-02	-1.1597E-03	5.8096E-02	358.86	4.23E-07	38
68.00	9.0464E-02	-1.3436E-03	9.0474E-02	359.15	5.46E-05	21
70.00	1.1592E-01	-1.5207E-03	1.1593E-01	359.25	2.83E-06	23
72.00	1.3384E-01	-1.6901E-03	1.3385E-01	359.28	5.53E-05	30
74.00	1.4265E-01	-1.8514E-03	1.4266E-01	359.26	4.11E-05	29
76.00	1.4196E-01	-2.0040E-03	1.4197E-01	359.19	3.28E-05	26
78.00	1.3223E-01	-2.1476E-03	1.3225E-01	359.07	1.42E-06	23
80.00	1.1374E-01	-2.2817E-03	1.1377E-01	358.85	5.76E-05	27
82.00	8.7937E-02	-2.4062E-03	8.7940E-02	358.43	5.96E-05	33
84.00	5.6493E-02	-2.5207E-03	5.6549E-02	357.45	4.29E-05	30
86.00	2.1265E-02	-2.6249E-03	2.1427E-02	352.96	2.85E-06	23
88.00	-1.5218E-02	-2.7188E-03	1.5459E-02	190.13	5.08E-05	43
90.00	-5.0909E-02	-2.8016E-03	5.0986E-02	183.15	9.78E-05	49
92.00	-8.2465E-02	-2.8736E-03	8.3514E-02	181.97	9.89E-05	35
94.00	-1.105[illegible]E-01	-2.9344E-03	1.1089E-01	181.52	5.51E-07	23
96.00	-1.3131E-01	-2.9839E-03	1.3134E-01	181.30	1.84E-05	30
98.00	-1.4353E-01	-3.0271E-03	1.4356E-01	181.21	3.65E-05	33
100.00	-1.4673E-01	-3.0491E-03	1.4676E-01	181.19	4.47E-05	27
102.00	-1.4058E-01	-3.0651E-03	1.4061E-01	181.25	1.33E-06	23
104.00	-1.2535E-01	-3.0704E-03	1.2539E-01	181.40	3.71E-05	26
106.00	-1.0199E-01	-3.0654E-03	1.0204E-01	181.72	5.79E-05	29
108.00	-7.1872E-02	-3.0508E-03	7.1937E-02	182.43	5.10E-05	35
110.00	-3.6741E-02	-3.0274E-03	3.6866E-02	184.71	8.89E-06	23
112.00	1.0895E-03	-2.9960E-03	3.1876E-03	289.97	6.65E-05	58

Fig. 8. (Continued)

114.00	3.9461E-02	-2.9576E-03	3.9571E-02	355.71	6.21E-07	30
116.00	7.5930E-02	-2.9133E-03	7.5986E-02	357.80	7.53E-05	28
118.00	1.0808E-01	-2.8641E-03	1.0811E-01	358.48	4.57E-06	23
120.00	1.3404E-01	-2.8113E-03	1.3407E-01	358.80	9.57E-05	30
122.00	1.5182E-01	-2.7558E-03	1.5185E-01	358.96	6.86E-05	31
124.00	1.6019E-01	-2.6987E-03	1.6021E-01	359.03	3.35E-05	19
126.00	1.5872E-01	-2.6409E-03	1.5874E-01	359.05	4.44E-06	23
128.00	1.4683E-01	-2.5833E-03	1.4685E-01	358.99	5.77E-05	31
130.00	1.2520E-01	-2.5265E-03	1.2522E-01	358.84	1.05E-06	32
132.00	9.4942E-02	-2.4711E-03	9.4974E-02	358.51	9.65E-05	33
134.00	5.7613E-02	-2.4174E-03	5.7664E-02	357.60	1.72E-05	23
136.00	1.5461E-02	-2.3656E-03	1.5641E-02	351.30	6.87E-05	53
138.00	-2.9162E-02	-2.3158E-03	2.9253E-02	184.54	2.45E-06	38
140.00	-7.3486E-02	-2.2683E-03	7.3521E-02	181.77	5.60E-05	31
142.00	-1.1474E-01	-2.2223E-03	1.1476E-01	181.11	1.24E-05	23
144.00	-1.5027E-01	-2.1777E-03	1.5029E-01	180.83	7.66E-05	29
146.00	-1.7750E-01	-2.1346E-03	1.7752E-01	180.69	8.71E-06	31
148.00	-1.9451E-01	-2.0927E-03	1.9452E-01	180.62	2.76E-05	16
150.00	-1.9909E-01	-2.0517E-03	1.9910E-01	180.59	5.86E-05	17
152.00	-1.9103E-01	-2.0114E-03	1.9104E-01	180.60	7.70E-06	31
154.00	-1.6949E-01	-1.9718E-03	1.6951E-01	180.67	4.87E-05	33
156.00	-1.3493E-01	-1.9328E-03	1.3494E-01	180.82	9.27E-05	29
158.00	-8.8300E-02	-1.8945E-03	8.8320E-02	181.23	3.97E-05	23
160.00	-3.1410E-02	-1.8573E-03	3.1465E-02	183.38	4.67E-05	43
162.00	3.3554E-02	-1.8214E-03	3.3603E-02	356.89	4.74E-05	28
164.00	1.0387E-01	-1.7873E-03	1.0388E-01	359.01	9.98E-05	31
166.00	1.7642E-01	-1.7554E-03	1.7643E-01	359.43	3.96E-05	23
168.00	2.4603E-01	-1.7263E-03	2.4803E-01	359.60	2.03E-05	27
170.00	3.1541E-01	-1.7004E-03	3.1541E-01	359.69	5.97E-05	31
172.00	3.7542E-01	-1.6785E-03	3.7543E-01	359.74	7.74E-05	36
174.00	4.2534E-01	-1.6608E-03	4.2534E-01	359.78	5.59E-05	23
176.00	4.6274E-01	-1.6479E-03	4.6275E-01	359.80	9.00E-05	31
178.00	4.6597E-01	-1.6400E-03	4.6597E-01	359.81	9.02E-05	42

Fig. 8. (Continued)

180.00	4.9382E-01	-1.6374E-03	4.9383E-01	359.81	9.58E-05	48

S H E L L   S U R F A C E   R A D I A L   V E L O C I T Y

PROBLEM TITLE - POINT-DRIVEN SPHERE

PROBLEM 1. - OUTWARD CONCENTRATED FORCE AT NORTH POLE.

KA= 2.86

FREQUENCY =5467.32 HZ

COLATITUDE(DEGREES)	VELOCITY		ABSOLUTE VALUE	PHASE(DEG)	ERROR	FLAG
0.00	4.3647E-03	-2.5140E-01	2.5144E-01	270.99	9.72E-05	85
2.00	4.3473E-03	-2.3887E-01	2.3891E-01	271.04	9.40E-05	56
4.00	4.2956E-03	-2.1115E-01	2.1119E-01	271.17	6.46E-05	34
6.00	4.2104E-03	-1.7347E-01	1.7352E-01	271.39	8.13E-05	50
8.00	4.0932E-03	-1.3306E-01	1.3312E-01	271.76	9.65E-05	39
10.00	3.9460E-03	-9.1606E-02	9.1691E-02	272.47	3.88E-05	32
12.00	3.7713E-03	-5.2487E-02	5.2622E-02	274.11	4.08E-05	42
14.00	3.5722E-03	-1.5305E-02	1.5716E-02	283.14	8.05E-05	62
16.00	3.3517E-03	1.7259E-02	1.7582E-02	79.01	1.06E-05	54
18.00	3.1136E-03	4.3877E-02	4.3987E-02	85.94	6.13E-05	28
20.00	2.8614E-03	6.5085E-02	6.5152E-02	87.48	5.88E-05	43
22.00	2.5989E-03	7.8534E-02	7.8577E-02	88.10	6.18E-05	23
24.00	2.3299E-03	8.6489E-02	8.6521E-02	88.46	4.82E-05	36
26.00	2.0581E-03	8.7523E-02	8.7547E-02	88.65	7.64E-05	40
28.00	1.7870E-03	8.1989E-02	8.2009E-02	88.75	3.36E-05	31
30.00	1.5199E-03	7.2253E-02	7.2269E-02	88.79	4.09E-05	23
32.00	1.2598E-03	5.5423E-02	5.5437E-02	88.70	9.24E-05	16
34.00	1.0094E-03	3.9457E-02	3.9470E-02	88.53	7.34E-05	31
36.00	7.7102E-04	1.9779E-02	1.9794E-02	87.77	8.07E-05	59
38.00	5.4658E-04	-2.7522E-04	6.1196E-04	333.27	0.	0
40.00	3.3766E-04	-1.9573E-02	1.9576E-02	270.99	7.94E-05	49
42.00	1.4544E-04	-3.6965E-02	3.6965E-02	270.23	4.46E-06	38
44.00	-2.9257E-05	-5.1371E-02	5.1371E-02	269.97	6.73E-05	53
46.00	-1.8598E-04	-6.1815E-02	6.1816E-02	269.83	2.23E-05	23

Fig. 8. (Continued)

48.00	-3.2454E-04	-6.8560E-02	6.8561E-02	269.73	3.76E-05	37
50.00	-4.4804E-04	-7.0689E-02	7.0690E-02	269.64	3.61E-06	32
52.00	-5.4785E-04	-6.8063E-02	6.8065E-02	269.54	7.32E-05	41
54.00	-6.3351E-04	-6.1579E-02	6.1583E-02	269.41	1.59E-05	23
56.00	-7.0273E-04	-5.0940E-02	5.0944E-02	269.21	5.77E-05	35
58.00	-7.5635E-04	-3.7407E-02	3.7414E-02	268.84	3.81E-05	37
60.00	-7.9531E-04	-2.1907E-02	2.1921E-02	267.92	9.15E-05	60
62.00	-8.2059E-04	-5.2839E-03	5.3472E-03	261.17	8.65E-05	52
64.00	-8.3323E-04	1.1337E-02	1.1368E-02	94.20	3.79E-05	42
66.00	-8.3426E-04	2.7042E-02	2.7055E-02	91.77	2.09E-06	38
68.00	-8.2481E-04	4.0697E-02	4.0706E-02	91.16	9.70E-05	45
70.00	-8.0585E-04	5.1470E-02	5.1476E-02	90.90	8.82E-06	23
72.00	-7.7846E-04	5.9135E-02	5.9140E-02	90.75	8.19E-05	40
74.00	-7.4363E-04	6.2785E-02	6.2789E-02	90.68	6.04E-05	34
76.00	-7.0237E-04	6.2381E-02	6.2385E-02	90.65	8.63E-05	33
78.00	-6.5563E-04	5.8079E-02	5.8082E-02	90.65	4.47E-06	23
80.00	-6.0438E-04	4.9894E-02	4.9898E-02	90.69	7.72E-05	36
82.00	-5.4952E-04	3.8629E-02	3.8633E-02	90.82	3.38E-05	46
84.00	-4.9194E-04	2.4965E-02	2.4969E-02	91.13	4.19E-05	45
86.00	-4.3251E-04	9.6223E-03	9.6320E-03	92.57	8.79E-06	23
88.00	-3.7203E-04	-6.1049E-03	6.1162E-03	266.51	5.69E-05	45
90.00	-3.1128E-04	-2.1590E-02	2.1592E-02	269.17	9.93E-05	85
92.00	-2.5099E-04	-3.5701E-02	3.5702E-02	269.60	5.74E-05	43
94.00	-1.9192E-04	-4.7503E-02	4.7504E-02	269.77	1.78E-06	23
96.00	-1.3436E-04	-5.6468E-02	5.6469E-02	269.86	7.79E-05	30
98.00	-7.9133E-05	-6.1877E-02	6.1877E-02	269.93	8.33E-05	35
100.00	-2.6574E-05	-6.3283E-02	6.3283E-02	269.98	6.09E-05	36
102.00	2.2977E-05	-6.0818E-02	6.0818E-02	270.02	4.27E-06	23
104.00	6.9271E-05	-5.4294E-02	5.4295E-02	270.07	9.92E-05	33
106.00	1.1216E-04	-4.4285E-02	4.4285E-02	270.15	8.56E-05	34
108.00	1.5158E-04	-3.1419E-02	3.1420E-02	270.28	7.01E-05	53
110.00	1.8757E-04	-1.6330E-02	1.6331E-02	270.66	2.78E-05	23
112.00	2.2025E-04	-1.7249E-04	2.7976E-04	321.93	0.	0

Fig. 8. (Continued)

114.00	2.4980E-04	1.6284E-02	1.6286E-02	89.12	3.48E-06	38
116.00	2.7647E-04	3.1888E-02	3.1889E-02	89.50	1.35E-05	42
118.00	3.0056E-04	4.5634E-02	4.5635E-02	89.62	1.50E-05	23
120.00	3.2238E-04	5.6797E-02	5.6798E-02	89.67	9.75E-05	45
122.00	3.4227E-04	6.4392E-02	6.4393E-02	89.70	2.22E-05	37
124.00	3.6056E-04	6.7871E-02	6.7872E-02	89.70	9.02E-05	19
126.00	3.7755E-04	6.7312E-02	6.7314E-02	89.68	1.49E-05	23
128.00	3.9354E-04	6.2100E-02	6.2102E-02	89.64	8.03E-05	41
130.00	4.0876E-04	5.2749E-02	5.2751E-02	89.56	4.84E-06	32
132.00	4.2339E-04	3.9695E-02	3.9697E-02	89.39	6.49E-05	37
134.00	4.3788E-04	2.3569E-02	2.3573E-02	88.94	5.84E-05	23
136.00	4.5139E-04	5.4323E-03	5.4510E-03	85.25	6.05E-05	65
138.00	4.6496E-04	-1.3806E-02	1.3814E-02	271.93	1.19E-05	38
140.00	4.7796E-04	-3.2915E-02	3.2919E-02	270.83	9.61E-05	40
142.00	4.9062E-04	-5.0649E-02	5.0651E-02	270.55	3.89E-05	23
144.00	5.0275E-04	-6.5998E-02	6.6000E-02	270.44	6.87E-05	44
146.00	5.1424E-04	-7.7756E-02	7.7758E-02	270.38	3.73E-05	31
148.00	5.2497E-04	-8.5174E-02	8.5175E-02	270.35	6.02E-05	16
150.00	5.3492E-04	-8.7169E-02	8.7170E-02	270.35	3.39E-05	23
152.00	5.4369E-04	-8.3626E-02	8.3628E-02	270.37	3.30E-05	31
154.00	5.5149E-04	-7.4388E-02	7.4390E-02	270.42	8.99E-05	40
156.00	5.5819E-04	-5.9535E-02	5.9538E-02	270.54	7.01E-05	36
158.00	5.6378E-04	-3.9504E-02	3.9508E-02	270.82	3.78E-05	56
160.00	5.6828E-04	-1.5030E-02	1.5041E-02	272.17	7.21E-05	61
162.00	5.7175E-04	1.2915E-02	1.2927E-02	87.47	5.19E-05	38
164.00	5.7429E-04	4.3141E-02	4.3145E-02	89.24	7.99E-05	43
166.00	5.7604E-04	7.4353E-02	7.4355E-02	89.56	2.52E-05	36
168.00	5.7712E-04	1.0514E-01	1.0514E-01	89.69	7.81E-05	27
170.00	5.7769E-04	1.3414E-01	1.3414E-01	89.75	2.56E-05	32
172.00	5.7791E-04	1.5996E-01	1.5996E-01	89.79	8.03E-05	39
174.00	5.7791E-04	1.8141E-01	1.8141E-01	89.82	7.78E-05	50
176.00	5.7782E-04	1.9752E-01	1.9752E-01	89.83	6.91E-05	34
178.00	5.7773E-04	2.0746E-01	2.0749E-01	89.84	9.46E-05	57

Fig. 8. (Continued)

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180.00          9.7769E-04    2.1088E-01    2.1088E-01       89.84       9.76E-05       90

KA VS. SHELL RADIAL VELOCITY AT NORTH POLE.

PROBLEM 1. - OUTWARD CONCENTRATED FORCE AT NORTH POLE.

    KA        POLAR VELOCITY
  2.0627E+00     2.9144E-01

INTERVAL = 1   TOTAL KA COMPUTED IS    1
```

Fig. 8. (Continued)

The velocity calculation also demonstrates the usefulness of the data parameter, NTERMS, for bounding the number of series terms. Although the smaller-magnitude results for colatitudes 38° and 112° are not fully converged, they are sufficiently converged at 90 terms, relative to neighboring values that they can be useful without further refinement. For the case of nonconvergence, ERROR could have been set to the last value obtained for the left member of Eq. 41, and FLAG equal to NTERMS. The zero values, on the other hand, distinguish more clearly such cases in the column listings.

The edit summarizing polar velocity versus ka remains from an earlier version of RADSPHERE, before the general capability for calculating at a single colatitude was introduced. This edit, in contrast to the general one, treats only velocities at colatitude 0°.

To plot the results stored internally from the RADSPHERE run, the additional data in Table 6 are supplied to PLOTTER.

Table 6. PLOTTER input for problem of point-driven spherical shell.

Data Item	Value
NKA	1
ORIG, RMAX, STEPSIZE (for surface pressure)	0., 13., 2.6
ORIG, RMAX, STEPSIZE (for surface velocity)	-0.26, 0.26, 0.104

The calculated absolute pressures have been multiplied by the shell radius squared for comparison with the published results. Figure 9 shows the echo of identification data which heads the data file being plotted. The data to be plotted on this file can also be echoed, if necessary, by using the flag NDIAGN in the RADSPHERE run.

```
DATA TO BE PLOTTED IS FROM THE PROGRAM RADSPHERE.

PROBLEM TITLE - POINT-DRIVEN SPHERE

LOADING CONDITION

     POINT LOAD
          MAGNITUDE     =  1.00

PROBLEM TYPES THAT WILL BE PLOTTED

               1

QUANTITIES AVAILABLE FOR PLOTTING

     SHELL SURFACE            SHELL SURFACE           FAR - FIELD
       VELOCITY                 PRESSURE                PRESSURE

          X                        X

DATA CALCULATED AT -

     INITIAL COLATITUDE =  0.00
     FINAL COLATITUDE   =180.00
     DELTA COLATITUDE   =  2.00

     DATA IS FOR  1 INTERVALS IN KA

     PLOTS TO BE MADE ARE POLAR TYPE


END OF DISSPLA 8.2 --     6049 VECTORS GENERATED IN     2 PLOT FRAMES.
-ISSCO-  4186 SORRENTO VALLEY BLVD.,SAN DIEGO CALIF. 92121

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   IS SUBJECT TO A NONDISSEMINATION AND NONDISCLOSURE AGREEMENT.
```

Fig. 9. Printer output from PLOTTER for point-driven spherical shell.

The RADSPHERE results for $a^2|\text{Surface Pressure}|$, Fig. 10, are generally in good agreement with Hayek's[3] results, Fig. 11, with respect to profile. To facilitate comparison, the graph in Fig. 10 has been labeled with respect to the coordinate system used for computing.

Figure 12 illustrates the method (see pages 19-20) used to plot shell surface radial velocities.

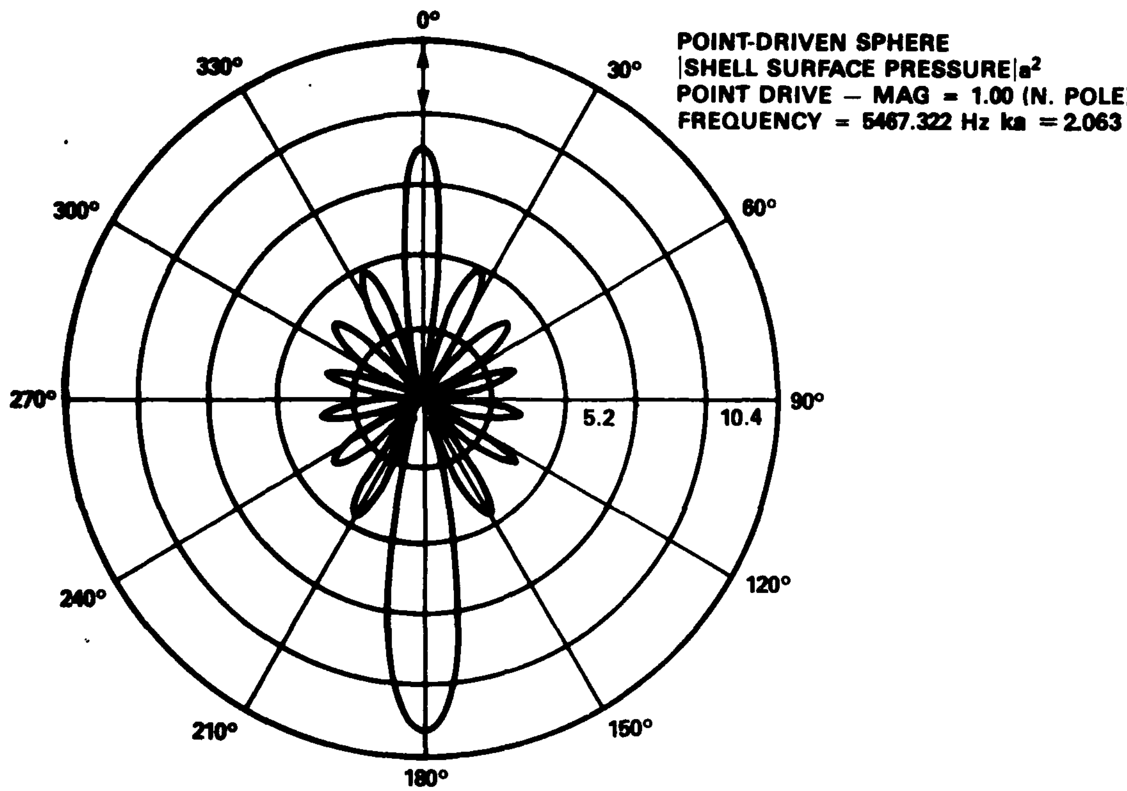

Fig. 10. RADSPHERE shell surface pressures for point-driven shell, $\Omega = 0.85$ and $h/a = 0.03$.

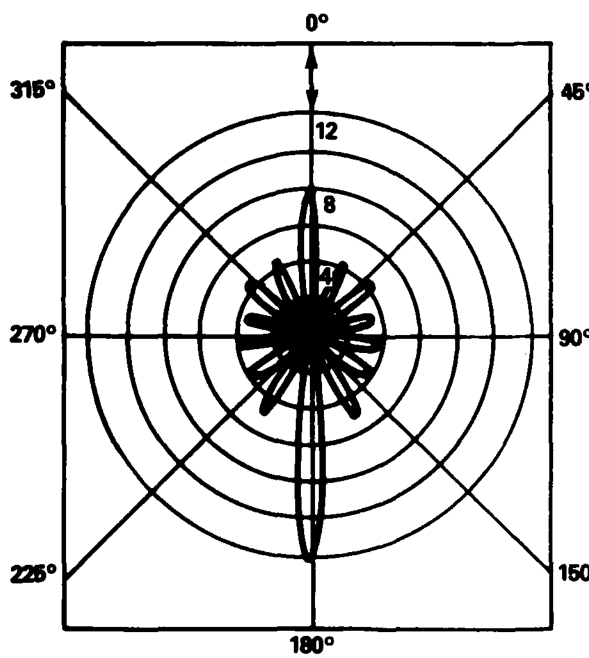

Fig. 11. Pressure on point-driven shell, $|P_a| a^2$, for $\Omega = 0.85$ and $h/a = 0.03$. (Reproduced from Hayek[3])

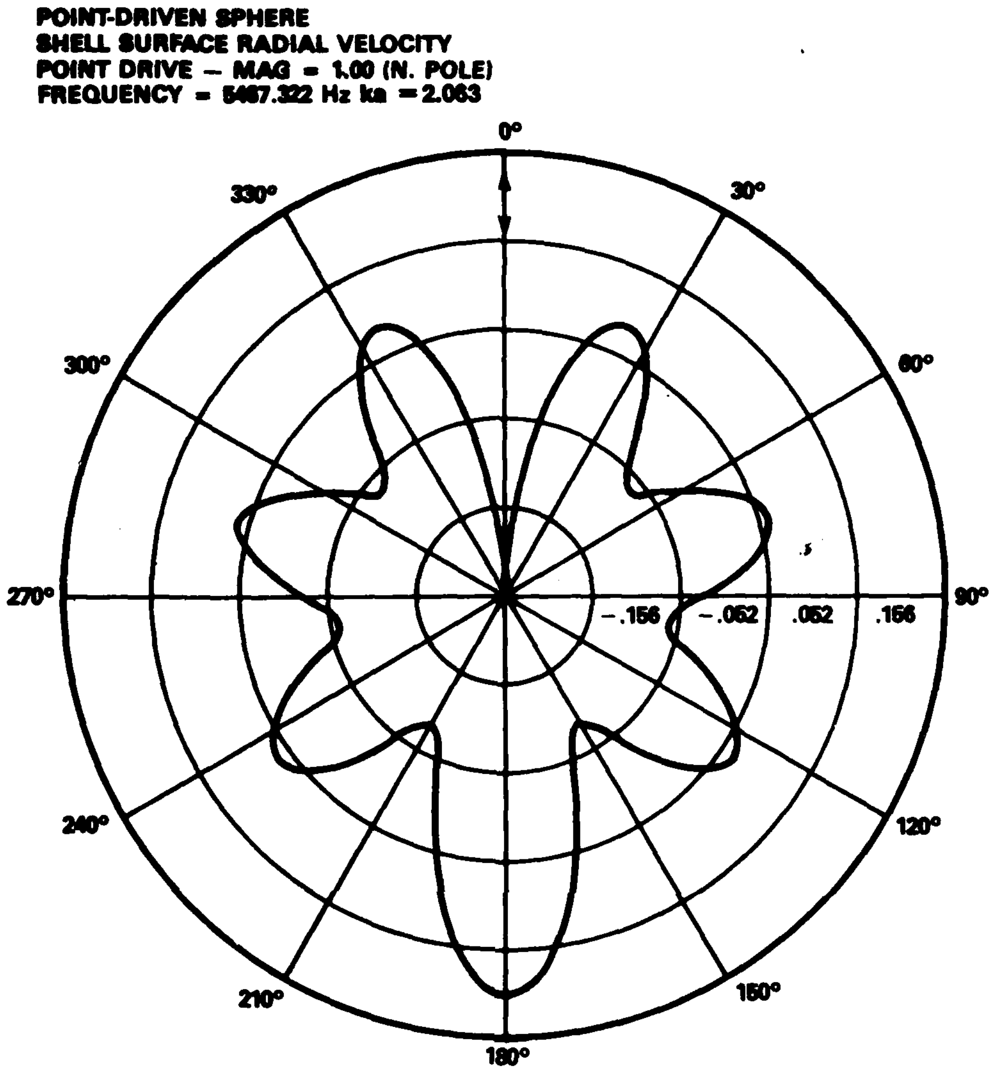

Fig. 12. Surface radial velocity for point-driven shell, $\Omega = 0.85$ and $h/a = 0.03$.

SECTOR-DRIVEN SPHERICAL SHELL

The third problem is from a set of calculations used in the validation of NASHUA. The shell is driven by a uniformly distributed load centered about the pole, $\theta = 0°$, Fig. 2, and having polar angle $\alpha = 36°$. Given the load magnitude, $F_0 = 20$, it is required to compute the field pressure at radius $R = 100$ for $ka = 5$. The data for RADSPHERE are given in Table 7.

Table 7. **RADSPHERE input for problem of sector-driven spherical shell.**

Data Item	Value
Title	Sector-Driven Sphere
NFLAGF	2
F_0	20.
α	36.
a	5.
h	0.15
ρ_s	7669.
ν	0.3
E	2.07E11
η	0.
ρ_{fluid}	1000.
c	1524.
NDIAGN	1
NPROB	1
NFLAG	3
NRADII	1
R	100.
NTERMS	40
ε	0.0001
θ_1	0.
θ_2	180.
$\Delta\theta$	2.
NINT	1
$(ka_1)_1$, $(ka_2)_1$, $(\Delta ka)_1$	5., 5., 1.

Data for the shell are in MKS units for steel. Computed results are given in Fig. 13.

```
        D       A       T       A               E       C       H       O

PROBLEM TITLE - SECTOR-DRIVEN SPHERE

DATA FOR SHELL

     RADIUS           =5.00
     THICKNESS        = .15
     MASS DENSITY     =7.6690E+03
     POISSON RATIO    = .30
     YOUNG S  MODULUS=2.0700E+11
     LOSS FACTOR      =0.00

DATA FOR EXCITATION

     SECTOR LOAD
          POLAR ANGLE = 36.00  DEGREES
          MAGNITUDE   = 20.00

DATA FOR FLUID

     MASS DENSITY   =1.000000E+03
     SPEED OF SOUND =1.524000E+03

DATA FOR FIELD

     NO. OF RADII = 1

          RADIUS( 1) =100.00

PROBLEM TYPES TO BE CALCULATED

               1

QUANTITIES TO BE COMPUTED

     SHELL SURFACE           SHELL SURFACE           FAR - FIELD
        VELOCITY                PRESSURE               PRESSURE

                                                          X

DIAGNOSTICS

     DO NOT PRINT

CALCULATIONS TO BE MADE FOR -

     INITIAL COLATITUDE =   0.00
     FINAL COLATITUDE    =160.00
     DELTA COLATITUDE    =   2.00
```

Fig. 13. RADSPHERE output for sector-driven spherical shell.

```
MAX. NO. OF TERMS FOR A SERIES = 40

CONVERGENCE CRITERION = 1.0000E-04



KA INTERVAL NUMBER IS    1

   INITIAL KA = 5.0000E+00
   FINAL KA   = 5.0000E+00
   DELTA KA   = 1.0000E+00
```

Fig. 13. (Continued)

FIELD PRESSURE AT RADIUS = 100.00

PROBLEM TITLE - SECTOR-DRIVEN SPHERE

PROBLEM 1. - OUTWARD DISTRIBUTED FORCE AT NORTH POLE.

KA= 5.00

FREQUENCY = 242.55 HZ

COLATITUDE(DEGREES)	PRESSURE		ABSOLUTE VALUE	PHASE(DEG)	ERROR	FLAG
0.00	5.0919E-01	-4.9547E-02	5.1160E-01	354.44	2.78E-05	14
2.00	5.0793E-01	-4.8437E-02	5.1023E-01	354.55	2.64E-05	14
4.00	5.0416E-01	-4.5149E-02	5.0618E-01	354.88	2.22E-05	14
6.00	4.9797E-01	-3.9802E-02	4.9955E-01	355.43	1.59E-05	14
8.00	4.8797E-01	-3.1720E-02	4.8900E-01	356.28	8.49E-05	12
10.00	4.7830E-01	-2.3478E-02	4.7888E-01	357.19	4.22E-05	12
12.00	4.6651E-01	-1.3857E-02	4.6672E-01	358.30	4.59E-07	12
14.00	4.5264E-01	-3.0991E-03	4.5265E-01	359.61	8.50E-05	11
16.00	4.3672E-01	6.4596E-03	4.3680E-01	1.11	6.84E-05	12
18.00	4.1884E-01	2.0548E-02	4.1935E-01	2.81	8.67E-05	12
20.00	3.9911E-01	3.2816E-02	4.0045E-01	4.70	9.17E-05	12
22.00	3.7764E-01	4.4920E-02	3.8030E-01	6.78	6.28E-05	12
24.00	3.5462E-01	5.6519E-02	3.5909E-01	9.06	6.09E-05	12
26.00	3.3024E-01	6.7261E-02	3.3702E-01	11.52	2.86E-05	12
28.00	3.0474E-01	7.6899E-02	3.1429E-01	14.16	1.02E-05	12
30.00	2.7938E-01	8.5083E-02	2.9109E-01	16.99	1.14E-05	11
32.00	2.5149E-01	9.1635E-02	2.6766E-01	20.02	8.72E-05	12
34.00	2.2548E-01	9.5669E-02	2.4494E-01	22.99	1.01E-05	14
36.00	1.9815E-01	9.8567E-02	2.2131E-01	26.45	3.07E-06	14
38.00	1.7112E-01	9.9505E-02	1.9795E-01	30.18	5.92E-06	14
40.00	1.4490E-01	9.8377E-02	1.7514E-01	34.17	8.97E-05	12
42.00	1.2016E-01	9.5011E-02	1.5320E-01	38.33	3.78E-05	12
44.00	9.6858E-02	8.9754E-02	1.3205E-01	42.82	3.52E-05	12
46.00	7.4158E-02	8.3377E-02	1.1158E-01	48.35	2.66E-05	14

Fig. 13. (Continued)

48.00	5.4424E-02	7.4846E-02	9.2541E-02	53.98	1.79E-05	14
50.00	3.6942E-02	6.4919E-02	7.4694E-02	60.36	3.65E-07	14
52.00	2.1794E-02	5.3885E-02	5.8125E-02	67.98	2.68E-05	14
54.00	8.9853E-03	4.2059E-02	4.3008E-02	77.94	6.43E-05	14
56.00	-1.5404E-03	2.9765E-02	2.9805E-02	92.96	5.11E-06	15
58.00	-9.8847E-03	1.7319E-02	1.9941E-02	119.72	2.72E-06	15
60.00	-1.6174E-02	5.0070E-03	1.6931E-02	162.80	3.26E-06	15
62.00	-2.0543E-02	-6.9218E-03	2.1677E-02	198.62	4.75E-05	14
64.00	-2.3120E-02	-1.8267E-02	2.9466E-02	218.31	1.68E-05	14
66.00	-2.4023E-02	-2.8878E-02	3.7564E-02	230.24	5.03E-05	14
68.00	-2.3354E-02	-3.8649E-02	4.5157E-02	238.86	6.32E-05	14
70.00	-2.1201E-02	-4.7515E-02	5.2031E-02	245.95	6.12E-05	14
72.00	-1.7652E-02	-5.5436E-02	5.8179E-02	252.34	4.66E-05	14
74.00	-1.1934E-02	-6.2492E-02	6.4014E-02	259.26	2.50E-05	12
76.00	-6.7352E-03	-6.8347E-02	6.8678E-02	264.37	7.24E-06	14
78.00	4.1070E-04	-7.3284E-02	7.3285E-02	270.32	1.31E-05	14
80.00	8.5010E-03	-7.7157E-02	7.7624E-02	276.29	2.63E-05	14
82.00	1.7377E-02	-7.9909E-02	8.1777E-02	282.27	3.61E-05	14
84.00	2.6867E-02	-8.1471E-02	8.5786E-02	288.25	3.57E-05	14
86.00	3.6793E-02	-8.1766E-02	8.9663E-02	294.23	2.79E-05	14
88.00	4.6168E-02	-8.0245E-02	9.2578E-02	299.91	9.12E-05	12
90.00	5.7270E-02	-7.8280E-02	9.6993E-02	306.19	2.08E-06	15
92.00	6.6693E-02	-7.3915E-02	9.9556E-02	312.06	6.46E-05	12
94.00	7.7563E-02	-6.9053E-02	1.0385E-01	318.32	2.41E-05	14
96.00	8.7314E-02	-6.2271E-02	1.0724E-01	324.50	2.85E-05	14
98.00	9.6645E-02	-5.4098E-02	1.1076E-01	330.76	2.66E-05	14
100.00	1.0544E-01	-4.4614E-02	1.1449E-01	337.07	1.92E-05	14
102.00	1.1358E-01	-3.3928E-02	1.1854E-01	343.37	6.09E-06	14
104.00	1.2162E-01	-2.2710E-02	1.2392E-01	349.44	5.71E-05	12
106.00	1.2815E-01	-1.0016E-02	1.2854E-01	355.53	1.25E-05	12
108.00	1.3320E-01	3.5163E-03	1.3325E-01	1.51	7.59E-05	12
110.00	1.3652E-01	1.7872E-02	1.3768E-01	7.46	2.31E-05	14
112.00	1.3904E-01	3.2203E-02	1.4272E-01	13.04	2.00E-05	14

Fig. 13. (Continued)

114.00	1.3958E-01	4.6686E-02	1.4756E-01	16.44	1.28E-05	14
116.00	1.3851E-01	6.1577E-02	1.5158E-01	23.97	4.71E-05	11
118.00	1.3597E-01	7.5781E-02	1.5567E-01	29.13	9.01E-05	12
120.00	1.3202E-01	8.9374E-02	1.5943E-01	34.10	3.68E-05	12
122.00	1.2679E-01	1.0211E-01	1.6279E-01	38.85	2.10E-05	12
124.00	1.2049E-01	1.1379E-01	1.6570E-01	43.37	7.46E-05	12
126.00	1.1322E-01	1.2427E-01	1.6811E-01	47.66	4.99E-05	13
128.00	1.0492E-01	1.3366E-01	1.6992E-01	51.87	9.16E-06	14
130.00	9.6306E-02	1.4163E-01	1.7127E-01	55.79	1.59E-07	14
132.00	8.7703E-02	1.4812E-01	1.7214E-01	59.37	9.60E-06	14
134.00	8.0450E-02	1.5251E-01	1.7243E-01	62.19	6.12E-05	12
136.00	7.2544E-02	1.5622E-01	1.7224E-01	65.09	2.70E-05	12
138.00	6.5206E-02	1.5876E-01	1.7163E-01	67.67	3.37E-05	12
140.00	5.8610E-02	1.6027E-01	1.7066E-01	69.91	9.21E-05	12
142.00	5.3300E-02	1.6071E-01	1.6931E-01	71.65	6.92E-06	14
144.00	4.8990E-02	1.6045E-01	1.6776E-01	73.02	4.05E-06	14
146.00	4.5566E-02	1.5979E-01	1.6616E-01	74.08	1.49E-05	14
148.00	4.2973E-02	1.5893E-01	1.6464E-01	74.87	2.31E-05	14
150.00	4.0074E-02	1.5872E-01	1.6370E-01	75.83	2.03E-05	11
152.00	3.9327E-02	1.5774E-01	1.6257E-01	76.00	1.97E-05	12
154.00	3.9435E-02	1.5667E-01	1.6175E-01	75.89	5.97E-05	12
156.00	3.9882E-02	1.5640E-01	1.6141E-01	75.69	4.08E-06	14
158.00	4.0750E-02	1.5628E-01	1.6151E-01	75.39	1.03E-05	14
160.00	4.2219E-02	1.5633E-01	1.6193E-01	74.89	2.36E-05	14
162.00	4.4239E-02	1.5649E-01	1.6263E-01	74.21	3.27E-05	14
164.00	4.6736E-02	1.5670E-01	1.6352E-01	73.39	3.52E-05	14
166.00	4.9595E-02	1.5691E-01	1.6456E-01	72.46	2.99E-05	14
168.00	5.3087E-02	1.5681E-01	1.6556E-01	71.30	1.30E-06	12
170.00	5.5766E-02	1.5716E-01	1.6676E-01	70.46	2.16E-06	14
172.00	5.8693E-02	1.5718E-01	1.6778E-01	69.52	2.47E-05	14
174.00	6.1239E-02	1.5716E-01	1.6867E-01	68.71	4.72E-05	14
176.00	6.3215E-02	1.5712E-01	1.6936E-01	68.08	6.64E-05	14
178.00	6.4467E-02	1.5709E-01	1.6980E-01	67.69	7.92E-05	14

Fig. 13. (Continued)

```
180.00          6.4995E-02   1.5707E-01     1.6995E-01        67.55        8.37E-05       14

INTERVAL = 1   TOTAL KA COMPLTED IS    1
```

Fig. 13. (Continued)

Data for PLOTTER to graph these results are in Table 8.

Table 8. PLOTTER input for problem of sector-driven spherical shell.

Data Item	Value
NKA	1
ORIG, RMAX, STEPSIZE	0., 0.60, 0.12

The problem identification echo from PLOTTER is shown in Fig. 14 and the plot in Fig. 15.

```
            DATA TO BE PLOTTED IS FROM THE PROGRAM RADSPHERE.

        PROBLEM TITLE - SECTOR-DRIVEN SPHERE

        LOADING CONDITION

            SECTOR LOAD
                POLAR ANGLE  = 36.00  DEGREES
                MAGNITUDE    = 20.00

        PROBLEM TYPES THAT WILL BE PLOTTED

                    1

        QUANTITIES AVAILABLE FOR PLOTTING

            SHELL SURFACE          SHELL SURFACE          FAR - FIELD
              VELOCITY               PRESSURE               PRESSURE

                                                               X

            NO. OF RADII FOR FIELD DATA ARE = 1
        DATA CALCULATED AT -

            INITIAL COLATITUDE =  0.00
            FINAL COLATITUDE   =180.00
            DELTA COLATITUDE   =  2.00

            DATA IS FOR  1 INTERVALS IN KA

            PLOTS TO BE MADE ARE POLAR TYPE


END OF DISSPLA 8.2 --      3192 VECTORS GENERATED IN      1 PLOT FRAMES.
-ISSCO-  4186 SORRENTO VALLEY BLVD.,SAN DIEGO CALIF. 92121

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   IS SUBJECT TO A NONDISSEMINATION AND NONDISCLOSURE AGREEMENT.
```

Fig. 14. Printer output from PLOTTER for sector-driven spherical shell.

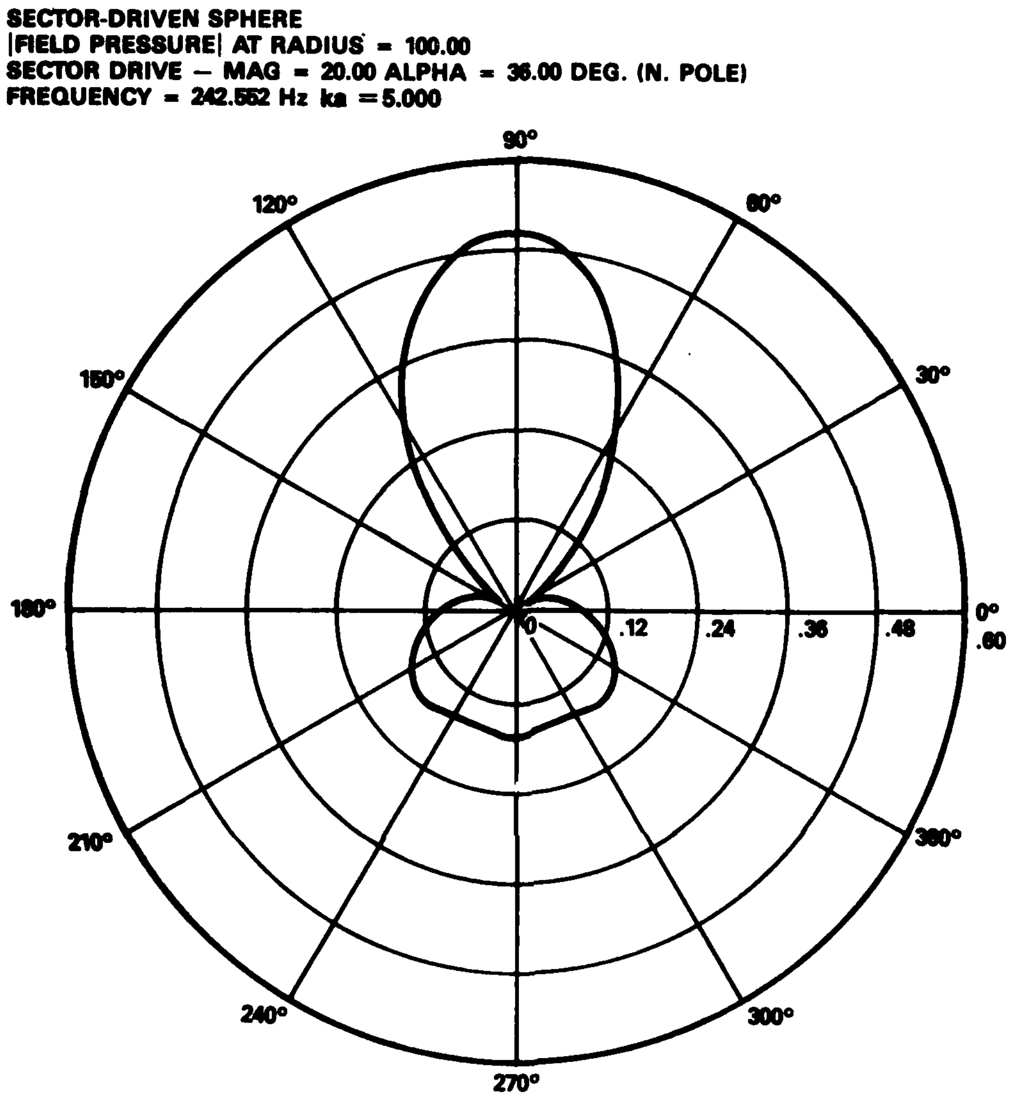

Fig. 15. RADSPHERE far-field pressure for sector-driven spherical shell.

Figure 16, a special purpose plot, shows superimposed the computed results from RADSPHERE and NASHUA[1] for this problem. Agreement is quite good.

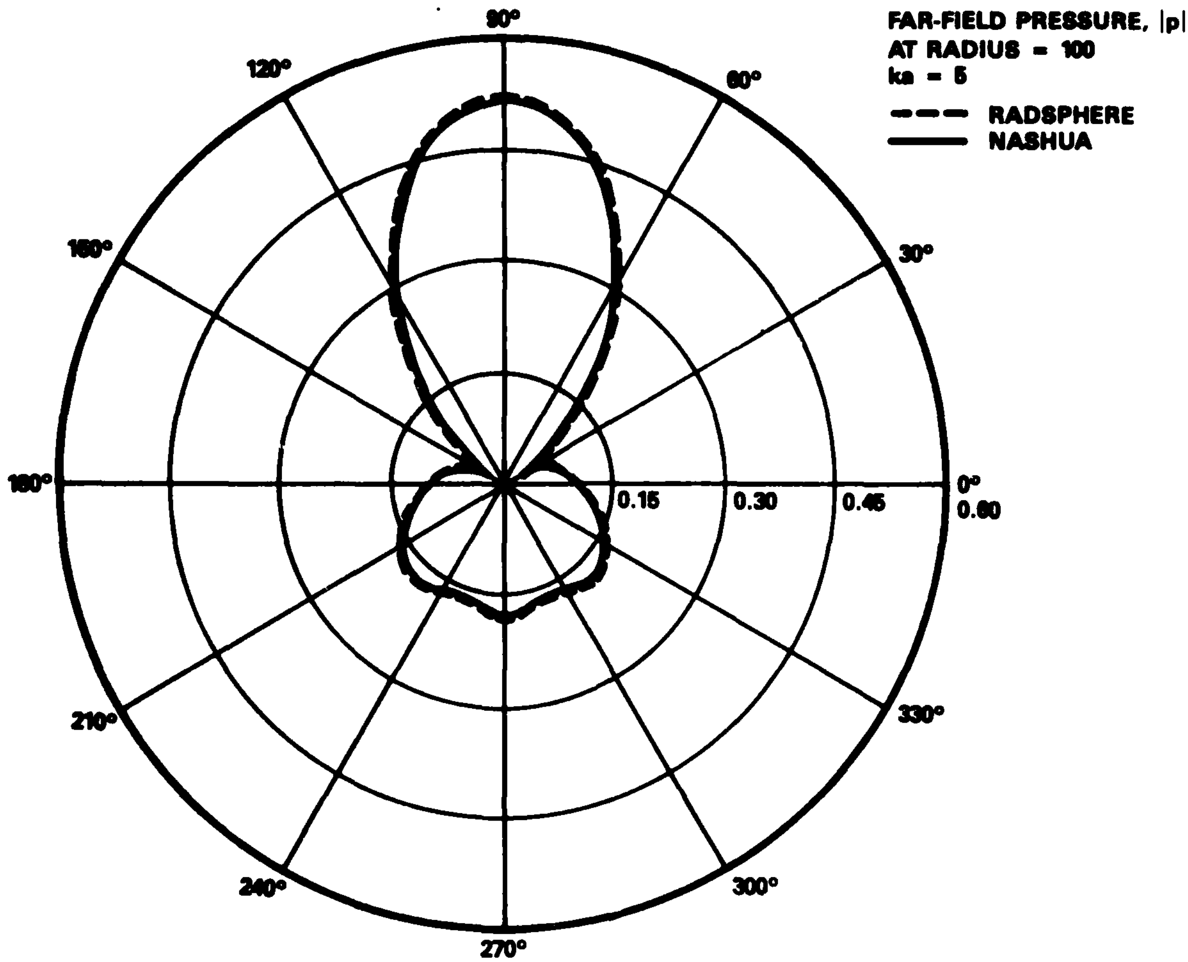

Fig. 16. Comparison of RADSPHERE and NASHUA's results for the sector-driven spherical shell.

RESONANCE SEARCH

In the course of validating NASHUA[1] for axisymmetric vibration of submerged spherical shells, having the resonant frequencies of both structural and field response was essential, so that those ka values could be determined at which the numerical calculations might be strongly influenced by such effects. Our fourth and final calculation demonstrates a search for resonances of the shell used in the previous calculation. Data to run the search were obtained by modifying the

data (in Table 7) for the sector-driven spherical shell. The modified data for this problem are shown in Table 9.

Table 9. RADSPHERE input for resonance search.

Data Item	Value
Title	Resonance Search
NFLAGF . . . NPROB	Same
NFLAG	5
NRADII R	Omit
NTERMS	60
ϵ θ_1	Same
θ_2	0.
$\Delta\theta$	1.
NINT	Same
$(ka_1)_1$, $(ka_2)_1$, $(\Delta ka)_1$	0.01, 10., 0.005

Figure 17 gives the data input echo from RADSPHERE, followed by an abbbreviated version of the rather lengthy computed results.

The appropriate plot for this type of calculation is y-log, x-linear. When the data for RADSPHERE specify $\theta_1 = \theta_2$ the program assumes that response versus ka is being computed and sets a flag for log plots on the file generated for PLOTTER.

The data supplied to PLOTTER are given in Table 10.

```
D   A   T   A       E   C   H   O

PROBLEM TITLE - RESONANCE SEARCH

DATA FOR SHELL

     RADIUS           =5.00
     THICKNESS        = .15
     MASS DENSITY     =7.6690E+03
     POISSON RATIO    = .30
     YOUNG S  MODULUS=2.0700E+11
     LOSS FACTOR      =0.00

DATA FOR EXCITATION

     SECTOR LOAD
          POLAR ANGLE = 36.00  DEGREES
          MAGNITUDE   = 20.00

DATA FOR FLUID

     MASS DENSITY   =1.000000E+03
     SPEED OF SOUND =1.524000E+03

PROBLEM TYPES TO BE CALCULATED

             1

QUANTITIES TO BE COMPUTED

     SHELL SURFACE          SHELL SURFACE          FAR - FIELD
       VELOCITY               PRESSURE               PRESSURE

          X

DIAGNOSTICS

     DO NOT PRINT

CALCULATIONS TO BE MADE FOR -

     INITIAL COLATITUDE =  0.00
     FINAL COLATITUDE   =  0.00
     DELTA COLATITUDE   =  1.00

MAX. NO. OF TERMS FOR A SERIES = 60

CONVERGENCE CRITERION = 1.0000E-04

KA INTERVAL NUMBER IS   1

   INITIAL KA = 1.0000E-02
   FINAL KA   = 1.0000E+01
   DELTA KA   = 5.0000E-03
```

Fig. 17. RADSPHERE output for resonance search.

S H E L L   S U R F A C E   R A D I A L   V E L O C I T Y

COLATITUDE(DEGREES) = 0.00

PROBLEM TITLE - RESONANCE SEARCH

PROBLEM 1. - OUTWARD DISTRIBUTED FORCE AT NORTH POLE.

KA	FREQUENCY(HZ)	VELOCITY		ABSOLUTE VALUE	PHASE(DEG)	ERROR	FLAG
.010	.485	6.0009E-11	2.8568E-04	2.8568E-04	90.00	7.94E-05	3
.015	.728	1.3501E-10	1.9037E-04	1.9037E-04	90.00	3.74E-05	6
.020	.970	2.4001E-10	1.4272E-04	1.4272E-04	90.00	6.66E-05	6
.025	1.213	3.7497E-10	1.1412E-04	1.1412E-04	90.00	5.52E-05	7
.030	1.455	5.3990E-10	9.5041E-05	9.5041E-05	90.00	7.96E-05	7
.035	1.698	7.3476E-10	8.1479E-05	8.1479E-05	90.00	8.40E-05	11
.040	1.940	9.5953E-10	7.1250E-05	7.1250E-05	90.00	4.18E-05	12
.045	2.183	1.2142E-09	6.3292E-05	6.3292E-05	90.00	5.30E-05	12
.050	2.426	1.4987E-09	5.6921E-05	5.6921E-05	90.00	6.55E-05	12
.055	2.668	1.8130E-09	5.1704E-05	5.1704E-05	90.00	7.93E-05	12
.060	2.911	2.1571E-09	4.7353E-05	4.7353E-05	90.00	9.45E-05	12
.065	3.153	2.5309E-09	4.3626E-05	4.3626E-05	90.00	8.52E-05	16
.070	3.396	2.9344E-09	4.0461E-05	4.0461E-05	90.00	9.89E-05	16
.075	3.638	3.3675E-09	3.7717E-05	3.7717E-05	89.99	3.93E-05	17
.080	3.881	3.8302E-09	3.5310E-05	3.5310E-05	89.99	4.48E-05	17
.085	4.123	4.3224E-09	3.3184E-05	3.3184E-05	89.99	5.06E-05	17
.090	4.366	4.8440E-09	3.1292E-05	3.1292E-05	89.99	5.68E-05	17
.095	4.608	5.3950E-09	2.9596E-05	2.9596E-05	89.99	6.34E-05	17
.100	4.851	5.9754E-09	2.8067E-05	2.8067E-05	89.99	7.04E-05	17
.105	5.094	6.5849E-09	2.6681E-05	2.6681E-05	89.99	7.78E-05	17
.110	5.336	7.2237E-09	2.5419E-05	2.5419E-05	89.98	8.55E-05	17
.115	5.579	7.8915E-09	2.4264E-05	2.4264E-05	89.98	9.37E-05	17
.120	5.821	8.5882E-09	2.3227E-05	2.3227E-05	89.98	9.60E-05	21
.125	6.064	9.3139E-09	2.2249E-05	2.2249E-05	89.98	3.55E-05	22
.130	6.306	1.0068E-08	2.1346E-05	2.1346E-05	89.97	3.85E-05	22

Fig. 17. (Continued)

.135	6.549	1.0852E-08	2.0507E-05	2.0507E-05	89.97	4.16E-05	22
.140	6.791	1.1663E-08	1.9727E-05	1.9727E-05	89.97	4.49E-05	22
.145	7.034	1.2504E-08	1.8999E-05	1.8999E-05	89.96	4.83E-05	22
.150	7.277	1.3372E-08	1.8318E-05	1.8318E-05	89.96	5.18E-05	22
.155	7.519	1.4269E-08	1.7679E-05	1.7679E-05	89.95	5.54E-05	22
.160	7.762	1.5195E-08	1.7078E-05	1.7078E-05	89.95	5.92E-05	22
.165	8.004	1.6148E-08	1.6513E-05	1.6513E-05	89.94	6.32E-05	22
.170	8.247	1.7129E-08	1.5979E-05	1.5979E-05	89.94	6.73E-05	22
.175	8.489	1.8130E-08	1.5474E-05	1.5474E-05	89.93	7.15E-05	22
.180	8.732	1.9174E-08	1.4996E-05	1.4996E-05	89.93	7.59E-05	22
.185	8.974	2.0238E-08	1.4542E-05	1.4542E-05	89.92	8.05E-05	22
.190	9.217	2.1330E-08	1.4111E-05	1.4111E-05	89.91	8.52E-05	22
.195	9.460	2.2449E-08	1.3701E-05	1.3701E-05	89.91	9.00E-05	22
.200	9.702	2.3595E-08	1.3310E-05	1.3310E-05	89.90	9.50E-05	22
.205	9.945	2.4768E-08	1.2923E-05	1.2923E-05	89.89	3.93E-05	27
.210	10.187	2.5967E-08	1.2566E-05	1.2566E-05	89.88	4.14E-05	27
.215	10.430	2.7194E-08	1.2225E-05	1.2225E-05	89.87	4.35E-05	27
.220	10.672	2.8447E-08	1.1897E-05	1.1897E-05	89.86	4.58E-05	27
.225	10.915	2.9726E-08	1.1584E-05	1.1584E-05	89.85	4.81E-05	27
.230	11.157	3.1031E-08	1.1282E-05	1.1282E-05	89.84	5.05E-05	27
.235	11.400	3.2363E-08	1.0993E-05	1.0993E-05	89.83	5.29E-05	27
.240	11.643	3.3720E-08	1.0714E-05	1.0714E-05	89.82	5.55E-05	27
.245	11.885	3.5103E-08	1.0446E-05	1.0446E-05	89.81	5.81E-05	27
.250	12.128	3.6512E-08	1.0187E-05	1.0188E-05	89.79	6.08E-05	27
.255	12.370	3.7945E-08	9.9380E-06	9.9381E-06	89.78	6.35E-05	27
.260	12.613	3.9404E-08	9.6971E-06	9.6972E-06	89.77	6.64E-05	27
.265	12.855	4.0888E-08	9.4643E-06	9.4644E-06	89.75	6.93E-05	27
.270	13.098	4.2397E-08	9.2392E-06	9.2393E-06	89.74	7.23E-05	27
.275	13.340	4.3930E-08	9.0213E-06	9.0214E-06	89.72	7.55E-05	27
.280	13.583	4.5488E-08	8.8102E-06	8.8103E-06	89.70	7.87E-05	27
.285	13.825	4.7070E-08	8.6056E-06	8.6057E-06	89.69	8.20E-05	27
.290	14.068	4.8676E-08	8.4071E-06	8.4073E-06	89.67	8.54E-05	27
:	:	:	:	:	:	:	:

Fig. 17. (Continued)

9.870	478.798	5.2358E-06	-3.3416E-05	3.3824E-05	278.90	9.74E-05	42
9.875	479.040	5.2175E-06	-3.0974E-05	3.1410E-05	279.56	6.16E-05	47
9.880	479.283	5.1991E-06	-2.8772E-05	2.9238E-05	280.24	6.62E-05	47
9.885	479.526	5.1806E-06	-2.6807E-05	2.7303E-05	280.94	7.09E-05	47
9.890	479.768	5.1620E-06	-2.5044E-05	2.5570E-05	281.65	7.57E-05	47
9.895	480.011	5.1433E-06	-2.3451E-05	2.4008E-05	282.37	8.07E-05	47
9.900	480.253	5.1245E-06	-2.2004E-05	2.2593E-05	283.11	8.58E-05	47
9.905	480.496	5.1057E-06	-2.0685E-05	2.1306E-05	283.87	9.11E-05	47
9.910	480.738	5.0868E-06	-1.9476E-05	2.0130E-05	284.64	9.64E-05	47
9.915	480.981	5.0678E-06	-1.8338E-05	1.9025E-05	285.45	6.36E-05	52
9.920	481.223	5.0487E-06	-1.7311E-05	1.8032E-05	286.26	6.71E-05	52
9.925	481.466	5.0297E-06	-1.6360E-05	1.7116E-05	287.09	7.07E-05	52
9.930	481.709	5.0105E-06	-1.5477E-05	1.6268E-05	287.94	7.45E-05	52
9.935	481.951	4.9913E-06	-1.4654E-05	1.5481E-05	288.81	7.83E-05	52
9.940	482.194	4.9721E-06	-1.3885E-05	1.4748E-05	289.70	8.22E-05	52
9.945	482.436	4.9529E-06	-1.3164E-05	1.4065E-05	290.62	8.63E-05	52
9.950	482.679	4.9336E-06	-1.2487E-05	1.3427E-05	291.56	9.04E-05	52
9.955	482.921	4.9143E-06	-1.1851E-05	1.2829E-05	292.52	9.47E-05	52
9.960	483.164	4.8950E-06	-1.1250E-05	1.2269E-05	293.51	9.90E-05	52
9.965	483.406	4.8757E-06	-1.0701E-05	1.1759E-05	294.50	6.74E-05	57
9.970	483.649	4.8564E-06	-1.0164E-05	1.1264E-05	295.54	7.04E-05	57
9.975	483.892	4.8371E-06	-9.6546E-06	1.0799E-05	296.61	7.35E-05	57
9.980	484.134	4.8177E-06	-9.1711E-06	1.0359E-05	297.71	7.67E-05	57
9.985	484.377	4.7984E-06	-8.7112E-06	9.9453E-06	298.85	7.99E-05	57
9.990	484.619	4.7791E-06	-8.2730E-06	9.5542E-06	300.01	8.32E-05	57
9.995	484.862	4.7598E-06	-7.8550E-06	9.1846E-06	301.21	8.66E-05	57
10.000	485.104	4.7406E-06	-7.4557E-06	8.8351E-06	302.45	9.01E-05	57

INTERVAL = 1   TOTAL KA COMPUTED IS 1999

Fig. 17. (Continued)

Table 10. PLOTTER input for resonance search.

Data Item	Value
NKA	1999
NCYCLE, YMIN	7, 1.E-7
XSTEP	1.
NXTICK, NYTICK	-2, -9

Figure 18 gives the echo from PLOTTER identifying the data, and Fig. 19 shows the response curve of radial velocity at $\theta = 0°$ versus ka, whose peaks indicate the resonances.

For this calculation the total ka computed is just two under the maximum currently allowed by the program. In the event that more resolution is needed, the primary interval can be subdivided and the same number of points used over each subinterval. Computation over the entire set of subintervals is done in a single run of RADSPHERE.

The 1999 points in the velocity response curve were calculated in 9.462 CP seconds on the Center's CDC 176 computer.

```
DATA TO BE PLOTTED IS FROM THE PROGRAM RADSPHERE.

PROBLEM TITLE - RESONANCE SEARCH

LOADING CONDITION

     SECTOR LOAD
          POLAR ANGLE = 36.00  DEGREES
          MAGNITUDE   = 20.00

PROBLEM TYPES THAT WILL BE PLOTTED

               1

QUANTITIES AVAILABLE FOR PLOTTING

     SHELL SURFACE            SHELL SURFACE            FAR - FIELD
        VELOCITY                 PRESSURE                 PRESSURE

          X

DATA CALCULATED AT -

     INITIAL COLATITUDE =  0.00
     FINAL COLATITUDE   =  0.00
     DELTA COLATITUDE   =  1.00

     DATA IS FOR  1 INTERVALS IN KA

     PLOTS TO BE MADE ARE LOG-LINEAR TYPE


END OF DISSPLA 8.2 --      3672 VECTORS GENERATED IN      1 PLOT FRAMES.
-ISSCO-  4186 SORRENTO VALLEY BLVD.,SAN DIEGO CALIF. 92121

DISSPLA IS A CONFIDENTIAL PROPRIETARY PRODUCT OF ISSCO AND ITS USE
   IS SUBJECT TO A NONDISSEMINATION AND NONDISCLOSURE AGREEMENT.
```

Fig. 18. Printer output from PLOTTER for resonance search.

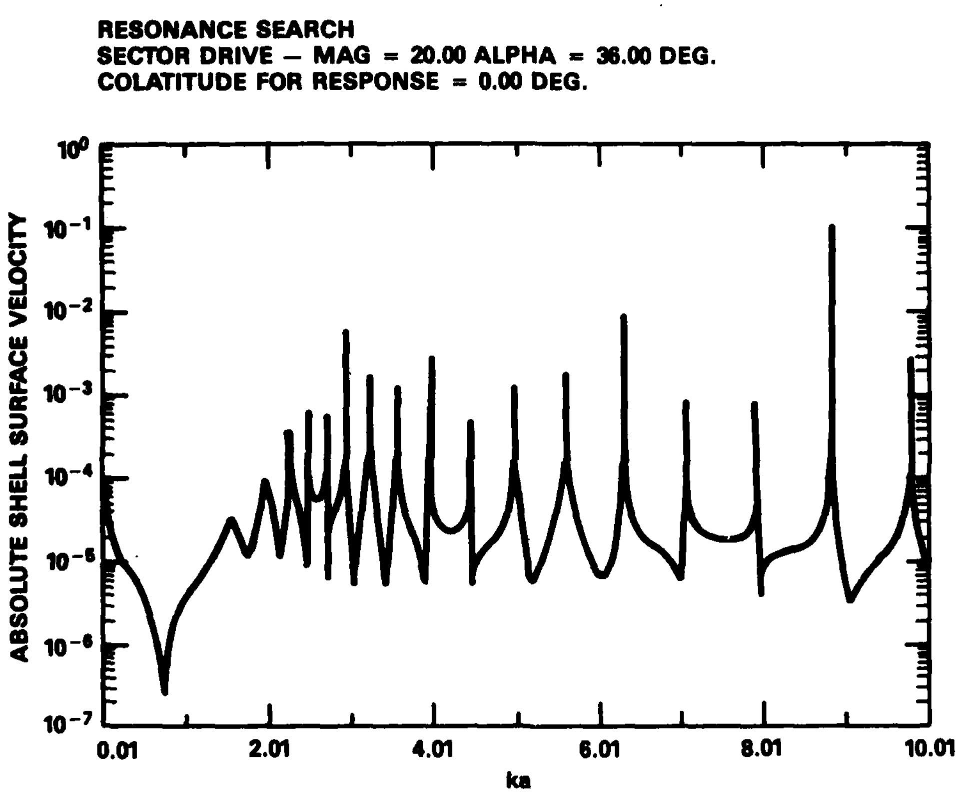

Fig. 19. Polar velocity versus ka for sector-driven spherical shell.

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REFERENCES

1. Everstine, G.C., F.M. Henderson, E.A. Schroeder, and R.R. Lipman, "A General Low Frequency Radiation Capability for NASTRAN," Fourteenth NASTRAN User's Colloquium, NASA CP-2419, pp. 293-310, National Aeronautics and Space Administration, Washington, DC (May 1986).

2. Junger, M.C. and D. Feit, Sound, Structures and Their Interactions, the MIT Press, Cambridge, Mass. (1972).

3. Hayek, S., "Vibration of a Spherical Shell in an Acoustic Medium," J. Acoust. Soc. A., Vol. 40, No. 2, pp. 342-348 (Feb 1966).

4. Wylie, C.R., Jr., Advanced Engineering Mathematics, second ed., McGraw-Hill, New York (1960).

5. IMSL (International Mathematics and Statistics Library), LIB-0009 (Ed 9.2), Vol. 3, Rev. (Nov 1984).

6. Abramowitz, M. and I. Stegun, editors, Handbook of Mathematical Functions, Dover Publications, Inc., N.Y. (Nov 1970).

7. Atkinson, K.E., An Introduction to Numerical Analysis, John Wiley and Sons, N.Y. (1978).

8. Control Data Corporation FORTRAN EXTENDED VERSION IV Reference Manual, pp. 5-7 to 5-10 (Jan 1981).

9. Smith, P.W., Jr. and R.H. Lyon, "Sound and Structural Vibration," NASA CR-160, National Aeronautics and Space Administration, Washington, D.C. (Mar 1965).

10. DISSPLA (Display Integrated Software System and Plotting Language), ISSCO, Visual Information System Software, User's Manual Version 10.0 (1981).

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